Creep is the progressive time dependent deformation of a material

Typical creep curve showing the three steps of creep. Solid line Constant load test. Dashed line constant stress test. The equation is Andrade’s empirical representation of the creep curve, $\varepsilon = \varepsilon_o \left( 1 + \beta t^{1/3} \right) e^{\kappa t}$, where $\varepsilon_o$, $\beta$, and $\kappa$ are constants.
Creep

There are a number of differing creep mechanisms; the operative or dominant one, usually determined by the level of applied stress and temperature which creep operates

There have been many engineering empirical approaches and equations developed to “predict” creep behavior as a function of stress level and time, but we will not discuss these.

In the 1970’s Mike Ashby developed so-called deformation mechanism maps which is a convenient way to categorize creep and begin our discussion.

Creep

The way in which \( \sigma_s \) (shear stress), \( T \) (temperature) and \( \dot{\gamma} \) (strain rate) are related for materials (a) when \( \sigma_s \) and \( T \) are prescribed and (b) when strain rate and \( T \) are prescribed, for low temperatures (top), high temperatures (middle) and very high temperatures (bottom).
Deformation mechanisms, divided into five groups.

Collapse at the ideal strength - (flow when the ideal shear strength is exceeded).

Low-temperature plasticity by dislocation glide - (a) limited by a lattice resistance (or Peierls' stress); (b) limited by discrete obstacles; (c) limited by phonon or other drags; and (d) influenced by adiabatic heating.

Low-temperature plasticity by twinning.

Power-law creep by dislocation glide, or glide-plus-climb — (a) limited by glide processes; (b) limited by lattice-diffusion controlled climb (“high-temperature creep”); (c) limited by corediffusion controlled climb (“low-temperature creep”); (d) power-law breakdown, (the transition from climb-plus-glide to glide alone); (e) Harper-Dorn creep; (f) creep accompanied by dynamic recrystallization.

Diffusional Flow -(a) limited by lattice diffusion (“Nabarro-Herring creep”); (b) limited by grain boundary diffusion (“Coble creep”); and (c) interface-reaction controlled diffusional flow.
Mechanisms of Creep Deformation

**Dislocation Glide:** This involves dislocation motion on slip planes over barriers by stress-assisted thermal activation. This mechanism dominates at high stress in the range, \( \sigma / \mu \geq 10^{-2} \)

Low-temperature plasticity limited by discrete obstacles. The strain-rate is determined by the kinetics of obstacle cutting.

\[
\gamma = \gamma_o \left[ -\frac{\Delta F}{kT} \left( 1 - \frac{\sigma_s}{\hat{\tau}} \right) \right]
\]

\( \Delta F \) is the total free energy (the *activation energy*) required to overcome the obstacle without aid from external stress.

\[
\gamma_o = \frac{\alpha}{b} \left( \frac{\sigma_s}{\mu} \right)^2 \beta \nu \approx 10^6 /s
\]

\( \hat{\tau} = \mu b / \sqrt{\rho} \) is the "athermal flow strength"— the shear strength in the absence of thermal energy.

\( \alpha \) and \( \beta \) are constants, \( \nu \) is an attempt frequency.
Mechanisms of Creep Deformation

Dislocation Glide

Low-temperature plasticity limited by a lattice resistance. The strain-rate is determined by the kinetics of kink nucleation and propagation.

\[ \gamma = \gamma_p \left( \frac{\sigma_s}{\mu} \right)^2 \exp \left\{ \frac{\Delta F_p}{kT} \left[ 1 - \left( \frac{\sigma_s}{\hat{\tau}} \right)^{3/4} \right]^{4/3} \right\} \]

Dislocation Creep – power-law creep

\[ \gamma = A \frac{D_{eff} \mu b}{kT} \left( \frac{\sigma_s}{\mu} \right)^n \]

\( n \) can vary anywhere between about 3 – 8.
Mechanisms of Creep Deformation

Diffusion Creep: Nabarro – Herring and Coble Creep

Nabarro – Herring Creep is controlled by lattice diffusion and Coble creep is controlled by grain boundary diffusion.

Nabarro-Herring creep and its rate scales as $D/d^2$. At lower temperatures, grain-boundary diffusion takes over; the flow is then called Coble creep, and scales as $D_b/d^3$.

$$\gamma = \frac{42\sigma_s \Omega}{kT d^2} D_{\text{eff}}$$

$$D_{\text{eff}} = D \left[ 1 + \frac{\pi \delta D_b}{d D} \right]$$

- $D$ - the lattice diffusivity
- $D_b$ - the GB diffusivity
- $\delta$ - the width of the GB
- $d$ - the grain size
- $\Omega$ - the atomic volume
The construction of a deformation-mechanism map. The field boundaries are the loci of points at which two mechanisms have equal rates.
Pure silver in grain size 1 mm.

Chromium with a grain size of 100 µm.
Mechanisms of Creep Deformation

Dislocation Glide

\[ \dot{\gamma} = \gamma_o \exp \left[ -\frac{\Delta G(\sigma - \sigma_I)}{k_B T} \right] = \gamma_o \exp \left[ -\frac{\Delta G(\sigma^*)}{k_B T} \right] = \gamma_o \exp \left[ -\frac{\Delta G(\tau^*)}{k_B T} \right] \]

\[ \dot{\gamma} = \text{strain rate} \]

\( \sigma = \text{stress} \)

\( \sigma_I = \text{internal stress} \) (due to work hardening, precipitates)

\( \tau^* \quad \sigma^* = \sigma - \sigma_I = \text{effective stress; friction stress} \)

\( \sigma = \sigma^* + \sigma_I = \text{flow stress} \)


\[
\dot{\gamma} = \dot{\gamma}_0 \exp \left[ - \frac{\Delta G^*(\tau^*)}{k_BT} \right]
\]

Thermally activated glide

Glide resistance,
\[\tau b\]
(force/length)

Reaction coordinate,
\[A = x \cdot l^*\]

\[
\Delta G(\tau^*)
\]

\[x_0 \quad x^* \quad x_0'\]
Orowan Equation

\[ \gamma = \rho b \lambda \]

\( \rho \) dislocation density

\( \lambda \) average distance moved by the dislocations

Taking the time derivative, the strain rate is given by,

\[ \dot{\gamma} = \rho b \frac{d\lambda}{dt} = \rho b v \]
Dislocation motion is thermally activated:

Velocity in  \[ \dot{\gamma} = \rho b \frac{d \lambda}{dt} = \rho b \nu \]

\[ \nu = l_0 \nu \exp\left(\frac{\Delta G^*}{k_B T}\right) = v_0 \exp\left(- \frac{\Delta G(\tau^*)}{k_B T}\right) \]

\( l_0 = \) distance moved by dislocation following activation
\( \nu = \) vibration frequency

\[ \dot{\gamma} = \rho b \nu_0 \exp\left(- \frac{\Delta G(\tau^*)}{k_B T}\right) \]
Activation energies for obstacles of strength, $f(x) = \text{constant}$

\[
\Delta G(\tau^*) = A^* b (\tau^*_0 - \tau^*) = A^* b \tau^*_0 \left(1 - \frac{\tau^*}{\tau^*_0}\right)
\]

\[
= \Delta G^*_0 \left(1 - \frac{\tau^*}{\tau^*_0}\right)
\]

$\tau_0^*$: stress required to overcome obstacle in absence of thermal activation ($T = 0K$)

$\tau^*$: stress in presence of thermal activation ($T > 0K$)
Activation energies for obstacles of strength, \( f(x) = \) parabola-shaped

\[
\Delta G(\tau^*) = \Delta G_0^* \left(1 - \frac{\tau^*}{\tau^*_0}\right)^{3/2}
\]
Temperature-dependent flow stress

\[ \dot{\varepsilon} = \dot{\varepsilon}_0 \exp \left[ -\frac{\Delta G(\sigma - \sigma_I)}{k_B T} \right] \]

\[ \Delta G(\tau^*) = \Delta G^* \left( 1 - \frac{\tau^*}{\tau_{0}^*} \right)^{3/2} \]

\[ \Delta G(\tau^*) = k_B T \ln(\dot{\varepsilon} / \dot{\varepsilon}_0) \]

\[ \sigma = \sigma_I + \sigma^*_0 \left[ 1 - \left( \frac{k_B T}{\Delta G^*_0} \ln(\dot{\varepsilon} / \dot{\varepsilon}_0) \right)^{2/3} \right] \]

athermal plateau: determined by the internal stress; obstacles that can’t be overcome by thermal activation
Effect of temperature on flow stress

Solid solution gives rise to both a friction (temperature-dependent) and internal (temperature independent) stress.

Note that flow stress of pure Ag depends weakly on T

Effect of Al concentration solid solution on flow stress in Ag-Al solid solution alloy.
Activation areas

\[ A^* \approx l^* \times x^* \]

\( l^* \) : activation length
\( x^* \) : activation distance (depends on the type of obstacle).
Measurement of $A^*$

$$\dot{\gamma} = \dot{\gamma}_0 \exp \left[ - \frac{\Delta G^* (\tau^*)}{k_B T} \right]$$

$$\frac{k_B T}{b} \left. \frac{\partial \ln(\dot{\gamma} / \dot{\gamma}_0)}{\partial \tau^*} \right|_{T, \gamma} = \frac{1}{b} \left. \frac{\partial \Delta G^*}{\partial \tau^*} \right|_{T, \gamma} \equiv A^*$$

$$A^* = \left. \frac{k_B T}{b} \frac{\partial \tau}{\partial \ln \dot{\varepsilon}} \right|_{T, \gamma}$$

Experimental measurement of $A^*$: Derivative of the flow stress with respect to strain rate
Measured activation areas

Haasen plot from nanoindentation on molybdenum thin films. In bcc metals the activation area varies little with flow stress.
Diffusion Creep: Nabarro – Herring and Coble Creep

The general equation describing the equilibrium vacancy concentration on planar surfaces of a stressed solid is,

\[
\Delta \mu = kT \ln \left( \frac{c_v}{c_v^o} \right) = -P\Omega \left( 1 + \frac{1-2\nu}{E} \sigma_{kk} \right) - \Omega \left[ -\frac{1}{2} \frac{\nu}{E} (\sigma_{kk})^2 + \frac{1+\nu}{2E} \sigma_{ij} \sigma_{ij} - \eta_v \sigma_{kk} \right]
\]

- \( c_v^o \) equilibrium vacancy concentration on the surface of an unstressed solid
- \(-P\) normal stress on the surface of interest
- \( \eta_v = \frac{\Delta \Omega}{3\Omega} \) vacancy relaxation
Consider a cylindrical bar loaded as shown below in uniaxial tension:

The surface under stress will have the vacancy formation energy reduced by an amount given by $\sigma_{zz} \Omega$.

$$\sigma_{zz} = -P$$

Consequently the equilibrium vacancy concentration on this surface will be given by

$$kT \ln \left( \frac{c_v}{c_v^o} \right) = \sigma_{zz} \Omega \left( 1 + \eta_v \right) + \frac{\sigma_{zz}^2}{2E} \Omega$$
Diffusion Creep: Nabarro – Herring and Coble Creep

The vacancy concentration on the traction free ($P = 0$) cylindrical surface,

$$\sigma_{rr} = \sigma_{\theta\theta} = 0$$

$$kT \ln \left( \frac{c_v}{c_v^o} \right) = \frac{\sigma_{zz}^2}{2E} \Omega + \eta_v \sigma_{zz} \Omega$$

When vacancy relaxation is neglected

$$kT \ln \left( \frac{c_v}{c_v^o} \right) = \sigma_{zz} \Omega + \frac{\sigma_{zz}^2}{2E} \Omega \quad \text{Loaded surface}$$

$$kT \ln \left( \frac{c_v}{c_v^o} \right) = \frac{\sigma_{zz}^2}{2E} \Omega \quad \text{Traction – free surface}$$
Diffusion Creep: Nabarro – Herring and Coble Creep

Einstein mobility relation, \( v = MF \), \( M = \frac{D}{kT} \); \( F = -\frac{\Delta \mu}{\Delta x} \)

The vacancy velocity

\[
v = -\frac{D}{kT} \frac{\Delta \mu}{\Delta x} = \frac{D}{kT} \frac{2\sigma\Omega}{L}
\]
Diffusion Creep: Nabarro – Herring and Coble Creep

This is related to the deformation of the bar.

\[ v = \frac{\Delta h}{\Delta t} = \frac{\Delta L}{\Delta t} = \frac{D}{kT} \frac{2\sigma \Omega}{L} \]

This can be converted to a strain rate by dividing by \( L \),

\[ \frac{\Delta L / L}{\Delta t} = \frac{\Delta \gamma}{\Delta t} = \frac{D}{kT} \frac{2\sigma \Omega}{L^2} \]

\[ \dot{\gamma} = \frac{D}{kT} \frac{2\sigma \Omega}{L^2} \quad \text{Nabarro Herring Creep} \]
Grain Boundary Diffusion

Diffusion along grain boundaries is more rapid than normal lattice diffusion.

Packing density in a grain boundary is less than the perfect lattice so atoms can change places more easily.
Isotope diffusion experiments indicate that

\[ \frac{D_{GB}}{D_L} \sim 10^6 \]

where \( Q_{GB} \approx 60KJ / \text{mole} \)

and \( Q_L \approx 100KJ / \text{mole} \)

When is GB diffusion important?

Depends upon GB width \( \delta \), grain size \( d \) and also \( D_{GB} / D_L \).

Consider the following model of steady state diffusion:

\[
\begin{align*}
\text{d} & \gg \delta \\
\delta & \approx 0.5 \text{nm}
\end{align*}
\]
Diffusion Creep: Nabarro – Herring and Coble Creep

Suppose there are $N_T$ atoms per unit time diffusing through a grain of width $d$ with boundary width $\delta$.

$$N_T = N_L + N_{GB}; \quad J_T = \frac{N_T}{(d + \delta)^2}; \quad J_L = \frac{N_L}{d^2}; \quad J_{GB} = \frac{N_{GB}}{\delta d}$$

$$\left(d + \delta\right)^2 J_T = d^2 J_L + d\delta J_{GB}$$

$$J_T \approx \frac{J_L d + J_{GB} \delta}{d}$$

from Fick’s 1st law – assuming steady state

$$J_T = -\left(\frac{D_L d + D_{GB} \delta}{d}\right) \frac{dC}{dx} = -D_{app} \frac{dC}{dx}$$
**Diffusion Creep: Nabarro – Herring and Coble Creep**

Defining an apparent diffusion coefficient

\[ D_{app} = D_L + D_{GB}\left(\frac{\delta}{d}\right) \]

or

\[ \frac{D_{app}}{D_L} = 1 + \frac{D_{GB}\delta}{D_Ld} \]

So obviously Grain boundary diffusion is important for

\[ D_{GB}\delta > D_Ld \]

Now recall

\[ \delta \sim 0.5\text{nm} \]

\[ d \sim 1 - 1000\mu m \]

So that

\[ \left(\frac{\delta}{d}\right) \sim 10^{-6} - 10^{-3} \]
At low temperature there is an important contribution of $D_{GB}$ to the flux. The cross-over occurs at $\sim 0.75 \ T_{mp}$. 
Diffusion Creep: Nabarro – Herring and Coble Creep

If we insert the equation for the apparent diffusion coefficient \( d \equiv L \) into the Nabarro-Herring creep equation for \( D \), we obtain,

\[
D_{app} = D_L + D_{GB} \left( \frac{\delta}{d} \right)
\]

\[
\dot{\gamma} = \frac{D_L}{kT} \frac{2\sigma\Omega}{L^2} \left( 1 + \frac{D_{GB} \delta}{D_L L} \right)
\]

Note that for Coble Creep (Grain Boundary Diffusion) the strain rate scales with \( 1/L^3 \) whereas for lattice diffusion it scales as \( 1/L^2 \).
Consider a "blunted " Mode I crack in plane stress undergoing a diffusively driven growth process. We are interested in calculating the steady state growth velocity of this crack.

Diffusional creep occurs owing to the difference in chemical potential between the crack tip surface and crack flank. This difference in chemical potential exists over the radius $\rho$ of the crack tip.

For a finite opening of crack, the crack surface is free of tractions. Recall that for a stressed surface, the chemical potential is
Creep Crack growth

\[ \Delta \mu = kT \ln \left( \frac{c_v}{c_v^o} \right) = -P \Omega \left( 1 + \frac{1 - 2\nu}{E} \sigma_{kk} \right) - \Omega \left[ -\frac{1}{2} \frac{\nu}{E} (\sigma_{kk})^2 + \frac{1 + \nu}{2E} \sigma_{ij} \sigma_{ij} - \eta_v \sigma_{kk} \right] \]

Neglecting lattice relaxation the equilibrium vacancy concentration on the crack surface at the crack tip, \( r = 0, \theta = 0 \) is,

\[ \Delta \mu = kT \ln \left( \frac{c_v}{c_v^o} \right) = -\frac{\sigma_{yy}^2}{2E} \Omega + \gamma \Omega K \]

\[ \text{Stress Effects} \quad \text{Curvature Term} \]

The second term on the rhs of this equation is a Gibbs-Thomson term. Crack growth occurs owing to the chemical potential difference between a vacancy at the crack tip and the unstressed crack flank. The equation describing the surface flux can be constructed either by consideration of the atom flux or vacancy flux. The difference in chemical potential for vacancies on the surface of the stressed crack gives rise to a flow of surface atoms with an average velocity described by the
Creep Crack growth

Einstein mobility relation,

\[ V_s = - \frac{D_s}{kT} \frac{d(\Delta \mu)}{ds} \]

Note that when \( \Delta \mu = 0 \),

\[ \Delta \mu = \frac{-\sigma_{yy}^2}{2E} \Omega + \gamma k \Omega = 0, \]

one obtains a Griffith-like equation for diffusive crack growth, \( \sigma_{yy} = \left(\frac{2E\gamma}{\rho}\right)^{1/2} \). For \( \sigma_{yy} < \left(\frac{2E\gamma}{\rho}\right)^{1/2} \) the crack will fill in with atoms owing to the capillary term.

Substituting for \( \Delta \mu \) in the mobility relation and multiplying the resultant equation by \( N_s \), the number of lattice sites per unit area, yield and equation for the surface flux, (# of vacancies per unit length per unit time)

\[ J_s = V_s N_s = -\frac{D_s N_s \Omega}{kT} \frac{d}{ds} \left( -\frac{\sigma_{yy}^2}{2E} + \gamma \kappa \right) \]
Creep Crack growth

The part of this equation involving the differential
\[
\frac{d}{ds} \left( -\frac{\sigma_{yy}^2}{2E} + \gamma \kappa \right),
\]
can be approximated by separately considering the difference in stress level between the crack tip and the crack flank \(\sigma_{yy} = 0; \kappa = 0\) over a distance \((\pi / 2) \rho\), which results in,

\[
J_s \equiv -\frac{D_s N_s \Omega}{kT} \quad \frac{2}{\pi} \quad \frac{1}{\rho} \left( -\frac{\sigma_{yy}^2}{2E} + \gamma \kappa \right)
\]

Conservation of matter requires that \(\rho V_n = J_s \Omega\) where \(V_n\) is the velocity of the crack. Making this substitution we obtain,

\[
V_n \equiv \frac{D_s N_s \Omega^2}{kT} \quad \frac{2}{\pi} \quad \frac{1}{\rho^2} \left( \frac{\sigma_{yy}^2}{2E} - \gamma \kappa \right)
\]