1. The figure below shows two parallel screw dislocations a distance $r$ apart. Each dislocation has a Burgers vector equal to $b\hat{k}$ and a tangent vector equal to $\hat{k}$, where $\hat{k}$ is the unit vector in the z-direction. Calculate the force/unit length that one dislocation exerts on the other.

![Diagram of two parallel screw dislocations](image)

2. If instead, the two parallel dislocations were edge dislocations each with tangent vector $\hat{k}$, and Burgers vector $b\hat{i}$ (where $\hat{i}$ is the unit vector in the x-direction) calculate the force/unit length that one dislocation exerts on the other.

3. If instead, the two parallel dislocations correspond to an edge and a screw, calculate the force/unit length that one dislocation exerts on the other. Note that both dislocations have a tangent vector equal to $\hat{k}$. For the edge dislocation the Burgers vector is $b\hat{i}$ and for the screw dislocation the Burgers vector is $b\hat{k}$.

4. If instead, the two parallel dislocations are of mixed character calculate the force/unit length that one dislocation exerts on the other. Note that both dislocations have a tangent vector equal to $\hat{k}$ and a Burgers vector equal to $\vec{b} = b_x\hat{i} + b_z\hat{k}$.

5. Consider a large thin plate of glass with a central crack of length $2a = 0.02$ m, Young’s modulus $E = 100$ GPa and a surface energy of $0.5$ Jm$^{-2}$. Calculate the Griffith fracture stress.
6. Consider a thin plate of glass containing an edge crack of length $a = 2$ mm as sketched below. The glass has a Young's modulus $E = 100$ GPa and a surface energy of $0.5$ Jm$^{-2}$. Calculate the critical value of the stress intensity factor for fracture of the glass plate.

7. A thin square plate made of elastically isotropic aluminum [$E = 72.0$ GPa, $\nu = 0.33$, $E = 2G(1+\nu)$] is subjected to the following stresses, $\sigma_{xx} = 250$ MPa, $\sigma_{yy} = -50$ MPa, and $\sigma_{xy} = -150$ MPa. If the yield strength of aluminum in simple tension is 350 MPa, determine if yielding occurs for the imposed multi-axial stress state.

**Additional problems for graduate students**

8. Consider an edge dislocation in a crystal. (a) Calculate the local volume change, $(\Delta V/V)$ around the dislocation as a function of $r$, $\theta$ by evaluation of the dilatational strain ($\varepsilon = \Delta V/V = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$) using the linear isotropic stress-strain relations and the stress field equations for the edge dislocation in polar coordinates. (b) Show that the total or net volume change associated with the edge dislocation in a crystal is zero. (c) Perform both calculation for a screw dislocation.

9. Consider the figure below, showing two parallel edge dislocations lying parallel to the z-axis passing through points $(0,0)$ and $(x_1, x_2)$ and having their Burgers vectors in the $x_1$ direction. (a) Calculate the glide component of the force per unit length that dislocation $\Delta$ exerts on $B$. (b) Determine the
value of the ratio, $x_1/x_2$ such that the dislocations are in a stable equilibrium position.

10. Consider a Mode I tensile crack (see sketch below) in a material with surface free energy, $\gamma = 2 \text{ J/m}^2$, and Young's modulus, $E = 3 \times 10^{11} \text{ Pa}$. There is a pure edge dislocation in the slip plane at a distance, $r$ from the crack tip equal to $1 \times 10^{-8} \text{ m}$. The pure edge dislocation has a tangent vector in the $z$ direction and Burgers vector, $b$, of magnitude $3 \times 10^{-10} \text{ m}$. The slip plane of the dislocation is oriented at an angle $\theta = 45^\circ$ with respect to the crack plane. There is a "frictional force per unit length" equal to $0.5 \text{ Nt./m} = 0.5 \text{ J/m}^2$ which must be overcome before the dislocation will move in its slip plane.

(a) Calculate the minimum value of the stress intensity $K_I$ that is required to get the dislocation to move on its slip plane. Hint: Use the mode I crack tip stress field equations in polar coordinates.

(b) Calculate the value of the stress intensity $K_I$ for brittle fracture of the material. (c) Will dislocation motion occur prior to brittle fracture of the solid?