Objectives:
- Use the plurality method of voting to determine a winner.
- Use the plurality with elimination method of voting to determine a winner.
- Use the instant run-off method of voting to determine a winner.
- Use the Borda count method of voting to determine a winner.
- Use the pairwise comparison method of voting to determine a winner.

Vocabulary:
- plurality
- majority

### Plurality Method of Voting

Each person votes for his or her favorite candidate (or choice). The candidate (or choice) who receives the most votes is declared the winner. (In case of a tie, special run-off elections may be held.)

### Plurality with Elimination Method of Voting

Each person votes for his or her favorite candidate (or choice). If a candidate receives a majority of votes, that candidate is declared the winner. If no candidate receives a majority, then the candidate with the fewest votes is eliminated and a new election is held. This process continues until a candidate receives a majority of the votes.
Instant Run-off Method of Voting

Each voter ranks all of the candidates; that is, each voter selects his or her first choice, second choice, third choice, and so on. If a candidate receives a majority of first-choice votes, that candidate is declared the winner. If no candidate receives a majority, then the candidate with the fewest first-choice votes is eliminated and those votes are given to the next preferred candidate. If a candidate now has a majority of first-choice votes, that candidate is declared the winner. If no candidate receives a majority, this process continues until a candidate receives a majority.

Borda Count Method of Voting

Each voter ranks all of the candidates; that is, each voter selects his or her first choice, second choice, third choice, and so on. If there are $k$ candidates, each candidate receives $k$ points for each first-choice vote, $(k-1)$ points for each second-choice vote, $(k-2)$ points for each third-choice vote, and so on. The candidate with the most total points is declared the winner.

Pairwise Comparison Method of Voting

Each voter ranks all of the candidates; that is, each voter selects his or her first choice, second choice, third choice, and so on. For each possible pairing of candidates, the candidate with the most votes receives 1 point; if there is a tie, each candidate receives $\frac{1}{2}$ point. The candidate who receives the most points is declared the winner.
Possible Classroom Examples:

Four candidates, Brecha, Parks, Wilcox, and Willett, are running for district supervisor. After the polls close, votes are tallied and the following results are obtained:

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Brecha</th>
<th>Parks</th>
<th>Wilcox</th>
<th>Willett</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Votes</td>
<td>3,007</td>
<td>2,957</td>
<td>10,541</td>
<td>2,851</td>
</tr>
</tbody>
</table>

a. How many votes were cast?
b. Using the plurality method of voting, which candidate wins?
c. Did the winner receive a majority of the votes?

The members of a local service club are volunteering to clean up and modernize the playground at one of the elementary schools in town. There are three schools: Hidden Lakes (H), Strandwood (S), and Valhalla (V). The members are asked to rank these choices in order of preference, and the results are summarized in the following voter preference table.

<table>
<thead>
<tr>
<th>Number of Ballots</th>
<th>Cast</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>19</td>
</tr>
</tbody>
</table>

| 1st choice | S | V | V | H |
| 2nd choice | H | S | H | S |
| 3rd choice | V | H | S | V |

a. How many votes were cast?
b. Use the plurality method of voting to determine the winner.
c. What percent of the votes did the winner in part (b) receive?
d. Use the instant run-off method to determine the winner.
e. What percent of the votes did the winner in part (d) receive?
f. Use the Borda count method to determine the winner.
g. How many points did the winner in part (f) receive?
h. Use the pairwise comparison method to determine the winner.
i. How many points did the winner in part (h) receive?

If there are eight candidates in an election and voters are asked to rank all of the candidates, how many different rankings are possible?

If there are eight candidates in an election and voters are asked to rank all of the candidates, how many different pairwise comparisons are there?
In an election, there are four candidates and 75 voters.

a. What is the maximum number of points that a candidate can receive using the Borda count method?

b. What is the minimum number of points that a candidate can receive using the Borda count method?