

Amortized Loan Example

Chris Columbus bought a house for \$293,000. He put 20% down and obtained a simple interest amortized loan for the balance at $5\frac{3}{8}\%$ annually interest for 30 years.

- Find the amount of Chris's monthly payment.
- Find the total interest paid by Chris.
- Most lenders will approve a home loan only if the total of all the borrower's monthly payments, including the home loan payment, is no more than 38% of the borrower's monthly income. How much must Chris make in order to qualify for the loan?
- Complete an amortization table for the first 2 months of the loan, the 180th through 181th months (only find the balance due for the 180th month) of the loan, and the final 2 months (only find the balance due for the 359th month) of the loan.

Solution:

- Our first step here is to find the down payment on the house. To calculate the down payment we need to multiply the price of the house by 20%.

$$\text{down payment} = 293000 * .20 = 58600$$

This means that Chris will be paying \$58,600 in cash for the house and financing the rest with an amortized loan.

Now we need to find the amount of the loan. This will be the difference between the price of the house and the down payment.

$$\text{loan amount} = 293000 - 58600 = 234400$$

Now we are ready to use the amortized loan formula with the loan amount (P), the annual interest rate ($r = .05375$), and the number of years of the loan ($n = 30$). This will give us

$$234400 \left(1 + \frac{.053875}{12} \right)^{(12*30)} = pymt \frac{\left(1 + \frac{.05375}{12} \right)^{(12*30)} - 1}{\left(\frac{.05375}{12} \right)}$$

$$1171369.056 = pymt(892.4223094)$$

$$\frac{1171369.056}{892.4223094} = pymt$$

$$1312.572584 = pymt$$

In this class we will round using standard rounding. This will make the monthly payment amount \$1312.57.

One thing that you should know about the monthly payment is that the payment does not all go to paying for the balance owed on the house. Some of every payment goes to paying the bank the interest that is accruing on the loan. During the early years of payment, the majority of the monthly payment goes to paying interest. We will see how much of each payment goes to interest and how much goes to paying off the loan in part d when we complete the amortization table.

- b. To find the total interest paid by Chris, we will use the formula $I = pymt * n * t - P$ (**Total Interest Formula for a Simple Interest Amortized Loan**) with Chris's loan amount and the monthly payment that we just calculated. This will give us

$$I = 1312.57 * 12 * 30 - 234400$$

$$I = 238125.20$$

- c. To answer this question, we have to make some assumptions. The biggest assumption that we need to make is that Chris has no other monthly expenses other than the monthly mortgage payment. We need the mortgage payment to be no more than 38% of Chris's monthly income. We can write this an equation that looks like

$$pymt = (\text{monthly income}) * .38$$

We can solve this equation for monthly income to find out the minimum monthly income allowed for the payment.

$$1312.57 = (\text{monthly income}) * .38$$

$$\frac{1312.57}{.38} = \text{monthly income}$$

$$3454.131579 = \text{monthly income}$$

Thus Gloria would have to have a minimum monthly income of \$3454.13 (and no other expenses) in order to qualify for this loan.

- d. Finally we have to work through an amortization table. An amortization table has a row for each month of the loan. The information in each row indicates the month of the loan, the principle portion of the loan payment, the interest portion of the loan payment, the total amount of the monthly payment, and the balance due on the loan.

Generally our amortization will look like the one below. Ours will skip a few rows to fit with the question.

Month	Principle Portion	Interest Portion	Total Monthly Payment	Balance Due on Loan
0				(a)
1	(b)	(c)		(d)
2	(e)	(f)		(g)
Skip Payments 2 through 179				
180				(h)
181	(i)	(j)		(k)
Skip Payments 182 through 359				
359				(l)
360	(m)	(n)		(o)

We will fill in the cells in the table with explanations as we go. Each cell that we fill in is labeled with a letter to make it easier to refer to. The table cell labeled (a) is where we place the amount of the loan. This is the balance due on the loan prior to making any payments. Thus the value for (a) is 234400.

The column labeled Total Monthly Payment is the monthly payment found in part a of this problem. This number will never change and that is why we do not label any of those table cells.

Table cell (c)

The next table cell that we can find is the interest portion of the first payment (labeled (c)). This is the interest on the balance of the loan for one month. For this we can use the formula $I = Prt$. In our problem P is the balance due on the loan ($P = 234400$). The interest rate is the same as that for the loan ($r = .05375$). The time for one payment is one month. Since t must always be in years, we must convert 1 month into years. This will give us $t = \frac{1 \text{ month}}{1} \times \frac{1 \text{ year}}{12 \text{ months}} = \frac{1}{12}$ years.

Now we plug all this into the formula to get

$$I = Prt = 234400(.05375)\left(\frac{1}{12}\right) = 1049.92$$
 (rounded to the nearest cent). Thus the cell marked (c) is 1049.95.

Table cell (b)

We can now find the value for the table cell labeled (b). Recall that the total monthly payment is made up of a part that pays down the principle and a part that pays for interest. Thus we can find the principle part by subtracting the interest portion from the total monthly payment (principle portion = monthly payment – interest portion). This will give us principle portion = $1312.57 - 1049.95 = 262.62$ for table cell (b).

Table cell (d)

Now we are ready to finish the row by calculating the table cell labeled (d). The balance due on the loan will decrease by the principle portion of the monthly payment. Thus we can find the new balance on the loan after the first monthly payment by new balance due = old balance due – principle portion. This will give us new balance due = $234400 - 262.62 = 234137.38$ for the table cell labeled (d).

Now the next row can be filled in the same way as the previous row. We will leave out the explanations.

Table cell (f):

P is the previous row's balance due on loan.

$$I = Prt = 234137.38(.05375)\left(\frac{1}{12}\right) = 1048.74$$

Table cell (e):

principle portion = montly payment – interest portion

$$\text{principle portion} = 1312.57 - 1048.74 = 263.83$$

Table cell (g):

new balance due = old balance due – principle portion

$$\text{new balance due} = 234137.38 - 263.83 = 233873.55$$

Now we are going to skip several rows of the amortization table. The next row that we need to fill something in for is row 180. What we need to fill in here is the balance due on the loan after 180 payments. You may have noticed that the principle portion of the monthly payment was going up a little bit with each payment. This mean the balance due on the loan went down by a different (and increasing) amount each month. Thus we cannot just subtract off the same number for 180 months to find the balance due.

In order to find the balance due, we need a new formula. This formula is the unpaid balance for an amortized loan formula

$$\text{unpaid balance} = P\left(1 + \frac{r}{n}\right)^{nT} - \text{pymt} \frac{\left(1 + \frac{r}{n}\right)^{nT} - 1}{\left(\frac{r}{n}\right)}$$

Where T is the number of years of payments on the loan.

Table cell (h):

We need to find T for 180 payments. We can do this by converting 180 months into years using dimensional analysis.

$$T = \frac{180 \text{ months}}{1} \times \frac{1 \text{ year}}{12 \text{ months}} = \frac{180}{12} \text{ years} = 15 \text{ years}$$

Now we plug everything into the formula

$$\text{unpaid balance} = 234400 \left(1 + \frac{.05375}{12}\right)^{12 \times 15} - 1312.57 \frac{\left(1 + \frac{.05375}{12}\right)^{12 \times 15} - 1}{\left(\frac{.05375}{12}\right)}$$

$$\text{unpaid balance} = 523993.2316 - 362039.5731$$

$$\text{unpaid balance} = 161953.6526$$

Rounding to a dollars and cents amount, we have that the balance due on the loan after 180 payments is 161953.65

Now the next row can be filled in the same way as previous rows. We will leave out the explanations.

Table cell (j):

P is the previous row's balance due on loan.

$$I = Prt = 161953.65(.05375)\left(\frac{1}{12}\right) = 725.42$$

Table cell (i):

principle portion = montly payment – interest portion

$$\text{principle portion} = 1312.57 - 725.42 = 587.15$$

Table cell (k):

new balance due = old balance due – principle portion

$$\text{new balance due} = 161953.65 - 587.15 = 161366.50$$

Again we are going to skip several rows to the 359th payment balance due. Again we will use the unpaid balance formula.

Table cell (l)

We need to find T for 359 payments. We can do this by converting 359 months into years using dimensional analysis.

$$T = \frac{359 \text{ months}}{1} \times \frac{1 \text{ year}}{12 \text{ months}} = \frac{359}{12} \text{ years}$$

Now we plug everything into the formula

$$\text{unpaid balance} = 234400 \left(1 + \frac{.05375}{12} \right)^{12 \times \left(\frac{359}{12} \right)} - 1312.57 \frac{\left(1 + \frac{.05375}{12} \right)^{12 \times \left(\frac{359}{12} \right)} - 1}{\left(\frac{.05375}{12} \right)}$$

$$\text{unpaid balance} = 1166145.695 - 1164836.683$$

$$\text{unpaid balance} = 1309.012418$$

Rounding to a dollars and cents amount, we have that the balance due on the loan after 359 payments is 1309.01.

Now the next row can be filled in the same way as previous rows. We will leave out the explanations.

Table cell (n):

P is the previous row's balance due on loan.

$$I = Prt = 1309.01(.05375) \left(\frac{1}{12} \right) = 5.86$$

Table cell (m):

principle portion = montly payment – interest portion

$$\text{principle portion} = 1312.57 - 5.86 = 1306.71$$

Table cell (o):

new balance due = old balance due – principle portion

$$\text{new balance due} = 1309.01 - 1306.71 = 2.30$$

You would expect that the balance due on the loan after the final loan payment would be 0. This is what happens in reality. The reason that it did not happen here is that when we rounded normally on our original payment amount, we rounded down. This means that we do not end up paying off the whole loan over the life of the loan. In real life, the payment amount would have been adjusted by a penny during the life of the loan so that the final payment would pay off the balance due on the loan before the final payment.

Month	Principle Portion	Interest Portion	Total Monthly Payment	Balance Due on Loan
0				(a) 234400
1	(b) 262.62	(c) 1049.95	1312.57	(d)234137.38
2	(e) 263.83	(f) 1048.74	1312.57	(g)233873.55
Skip Payments 2 through 179				
180				(h)161953.65
181	(i) 587.15	(j) 725.42	1312.57	(k)161366.50
Skip Payments 182 through 356				
359				(l) 1309.01
360	(m) 1306.71	(n) 5.86	1312.57	(o) 2.30