MAT 142 College Mathematics Module #4

Finance

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1. Simple Interest

Interest is the money earned (profit) on a savings account or investment. ***Principal*** or ***present value*** is the amount of money invested, sometimes referred to as the ***initial amount***.

Simple interest is when the money earned is computed as a percentage of the principal per year. The interest ***rate*** is the decimal equivalent of the percentage that will be earned.

Simple Interest Formula:

$$I = Prt$$

where I is the interest, P is the principal, r is the rate, and t is the time in years.

Example 1. Calculate the interest for a deposit of \$850 into an account paying 3.5% annual simple interest if the money is in the account for 7 months.

Solution: We are given $P = 850$ and $r = 0.035$, since there are 12 months in a year and the money will be in the account for 7 months, $t = 7/12$. So the interest will be:

$$I = Prt \quad \Longrightarrow \quad I = 850(0.035)\left(\frac{7}{12}\right) \approx \$7.35.$$

We round our answer to two decimal places since this is money.

At the end of the time, the total amount, principal and interest, is called the ***future value*** or ***maturity value***. There are two ways to compute this value.

Future Value for Simple Interest Formula:

$$FV = P + I \quad \text{or} \quad FV = P(1 + rt)$$

where I is the interest, P is the principal, r is the rate, and t is the time in years.

Example 2. What is the future value of a savings account earning $3\frac{1}{2}\%$ simple interest, if the present value is \$538 and the money is in the account for 7 months?

Solution:

(1) The first method is to compute the interest and then add that to the principal. We are given $P = 538$, $r = 0.035$, and $t = \frac{7}{12}$.

$$I = 538(0.035)\left(\frac{7}{12}\right) = 10.98 \qquad \Longrightarrow \qquad FV = 538 + 10.98 = \$548.98.$$

(2) The second method is to compute the future value directly.

$$FV = 538(1 + 0.035\left(\frac{1}{12}\right) = \$548.98.$$

Simple interest is used as the basis for other types of interest. The most common application of simple interest is called an add-on loan. An ***add-on loan*** is a loan in which the future value of the loan is calculated and then payments are determined by dividing this by the number of payments to be made. The following example demonstrates this type of loan.

Example 3. The Perez family buys a bedroom set at Mor Furniture for \$3,700. They put \$500 down and finance the rest through the store at 9.8% add-on interest. If they agree to make 36 monthly payments, find the size of each payment.

Solution: This is a simple interest problem. The interest is determined at the simple interest rate and added on to the amount of the loan. Our first step in the problem is to determine the amount of the loan. Since \$500 is used as a down payment, the amount of the loan will be $\$3700 - \$500 = \$3200$. Now we need to figure out what formula we will need to use. Remember that the amount of the loan plus the interest is the future value of the loan. Thus we will need to use the future value simple interest formula of $FV = P(1 + rt)$. In our problem P is the amount of the loan or \$3200. The interest rate is 9.8%. We need this to be changed from a percent to a decimal. To do this, we divide 9.8 by 100 to get 0.098. t is the time in years. Since the Perez family will be paying for 36 months, t will be 3.

$$FV = 3200(1 + 0.098 * 3).$$

Now we will plug all this in to our calculator to get

$$FV = 4140.80.$$

We now know how much the Perez family will pay in total for the bedroom set. We still have to figure out how much this will cost them in monthly payments. To do this, we will need to divide the future value by the number of month. This will give us

$$pymt = \frac{4140.80}{36} = 115.02222222.$$

Since we are talking about an amount of money, we must have only dollars and cents (two places to the right of the decimal). In this class we will round using standard rounding. This will make the payment amount \$115.02.

2. Compound Interest

Interest is ***compounded*** when the interest eaned for a specified time period is added into the account and then it also earns interest. Here is a simple example of how it works.

Example 4. George makes a deposit of \$50 into an account that earns 24% per year compounded monthly. He will leave the money in the account for 3 months. How much will he have at the end of three months.

Solution: Since the interest is 24% per year, at the end of the first month, George will get $50(0.24)(1/12) = \$1.00$ in interest (simple interest formula) deposited into the account. This increases his balamce to \$51 At the end of the second month, he will get $51(0.24)(1/12) = \$1.02$ in interest deposited into the account. His balance increases again to \$52.02. At the end of the third month, he will have another $52.02(0.24)(1/12) = \$1.04$ deposited into his account, so that when he draws out his money he will get \$53.06.

It should be pretty clear that we do not want to build this up one month at a time whenever we need to find the amount of an account earning compound interest. Hence. we use a formula but first we need to know another term.

The ***compounding period*** is the length of time that elapses before a new interest is deposited into the account. In our Example 4, the compounding period is one month or monthly.

Compound Interest Formula:

$$FV = P\left(1 + \frac{r}{n}\right)^{(nt)}$$

where FV is the future value, P is the principal, r is the rate, t is the time in years and n is the number of compounding periods per year.

Example 5. When Jacob was born, his grandparents deposited \$10,000 into a special account for Jacobs college education. The account earned $6\frac{1}{4}$ % interest compounded daily.

(1) How much will be in the account when Jacob is eighteen?
(2) If, on becoming eighteen, Jacob arranges for the monthly interest to be sent to him, how much would he receive each 30-day month?

Solution:

(1) The first part of this problem is a basic future value of a compound interest account question. For this, we will need the future value formula for compound interest $FV = P\left(1 + \frac{r}{n}\right)^{(nt)}$. For our problem P is 10000; r is 0.0625 (obtained by calculating $(6 + 1 \div 4) \div 100)$; n is 365 (number of days in a year); and t is 18 (number of years from Jacob's birth to age 18). We now plug these numbers into the formula to find the future value.

$$FV = 10000\left(1 + \frac{0.0625}{365}\right)^{(365 \cdot 18)} = 30799.20215.$$

Since we are talking about an amount of money, we must have only dollars and cents (two places to the right of the decimal). In this class we will round using standard rounding. This will make the future value \$30799.20.

(2) For this part of the problem, we only need to determine how much interest in earned in a total of 30 days. To determine the interest, we use the formula $FV = P + I$. The part that we need to realize here is that the principal that we are determining the interest on is the amount in Jacob's account when he turns 18. We will use the same formula that we used in part (1). For this part though, our P is 30799.20; r remains the same as in part (1); and n is still 365. Another thing that is different about this part of the problem is that nt will not be used here. In the previous problem nt produced the total number of compounding periods over the 18 years. In this case there is not even one year of compounding periods. Since we are asked about a 30-day month, there will only be 30 periods. Thus nt will be 30 for this part of the problem. When we plug this into the formula, we get

$$FV = 30799.20\left(1 + \frac{0.0625}{365}\right)^{30} = 30957.80853.$$

Since we are talking about an amount of money, we must have only dollars and cents (two places to the right of the decimal). In this class we will round using standard rounding. This will make the future value \$30957.81. Now we need to take the future value and subtract the principal to find the amount of interest earned. This will give us

$$I = 30957.81 - 30799.20 = 158.61.$$

The interest that Jacob gets each month will be \$158.61.

Example 6. David wants to have a retirement account that will be worth \$150,000 when he retires at age sixty-five.

(1) How much must he deposit at age twenty-six at $6\frac{3}{8}\%$ compounded daily?
(2) If, at age sixty-five, he arranges for the monthly interest to be sent to him, how much will he receive each 30-day month?

Solution:

(1) For this example, we need to find out how much to put into the account now (present value) to get a future value of \$150000 in a prescribed number of years. We will use the future value compound interest formula for this. In this problem, we have a FV of 150000; r is 0.06375 $\left(\text{calculated by } \frac{6+\frac{3}{8}}{100}\right)$; n is 365; and t is 39 (65-29). We will now be looking for P.

$$\begin{aligned}
150000 &= P\left(1 + \frac{0.06375}{365}\right)^{(365*39)} \\
150000 &= P(12.01352266) \\
\frac{150000}{12.01352266} &= P \\
P &= 12485.92975
\end{aligned}$$

For this problem, it is important that instead of using standard rounding, we round up to the next cent. If we rounded down, we would not quite get to \$150000. This will mean that David will need to deposit \$12,485.93 now.

(2) For this part of this problem, we are once again being asked to find how much interest is earned in 30 days. Just like in the previous example, we will need to find the future value of David's investment after 30 days once he turns 65. For this problem, our principal (the value when David turns 65) is given in the problem at 150000. r is 0.06372; and n is 365. Once again in this problem there is not even one year of compounding periods. Since we are asked about a 30-day month, there will only be 30 periods. Thus nt will be 30 for this part of the problem. When we plug this into the formula, we get

$$FV = 150000\left(1 + \frac{0.06375}{365}\right)^{30} = 150787.9526.$$

Since we are talking about an amount of money, we must have only dollars and cents (two places to the right of the decimal). In this class we will round using standard rounding. This will make the future value \$150787.95.

Now we need to take the future value and subtract the principal to find the amount of interest earned. This will give us

$$I = 150787.95 - 150000 = 787.95$$

The interest that David gets each month will be \$787.95.

Our final concept using compound interest is called the annual yield. The ***annual yield*** is the simple interest rate that will result in the same amount of interest in one year.

Annual Yield Formula:

$$ann\ yield = \left(1 + \tfrac{r}{n}\right)^n - 1$$

where r is the rate and n is the number of compounding periods per year

Example 7. Charleston Bank has a CD (certificate of deposit) that offers 5.62% annual interest compounded quarterly. Richardson Credit Union has a CD that offers 5.6% annual interest compounded weekly. Which CD is the better choice?

Solution: Since the interest rates and compounding periods are different, we will use the annual yield for each CD to compare them. For Charleston Bank we are given $r = 0.056$ and $n = 4$.

$$ann\ yield = \left(1 + \frac{0.0562}{4}\right)^4 - 1 \approx 0.0574, \quad \text{or } 5.74\%$$

For Richardson Credit Union $r = 0.054$ and $n = 52$.

$$ann\ yield = \left(1 + \frac{0.054}{52}\right)^{52} - 1 \approx 0.0576, \quad \text{or } 5.76\%$$

Since the credit union has a higher annual yield, it is the better investment choice.

An ***annuity*** is defined by merriam-webster.com as a sum of money payable yearly or at other regular intervals. Wikipedia defines an annuity as any recurring periodic series of payment. Some examples of annuities are regular payments into a savings account, monthly mortgage payments (to be dealt with seperately), regular insurance payments, etc. Annuities can be classified by when the payments are made. Annuities whose payments are made at the end of the period are called ***ordinary annuities***. Annuities whose payments are made at the beginning of the period are called ***annuity-due***. In this class we will only work with ordinary annuities.

We will need to be able to calculate the future value of our annuities. In order to do this we will need a formula to calculate future value if we know the amount of the payment, the interest rate, compounding period and time the annuity will be active.

Ordinary Annuity Formula:

$$FV = pymt\left[\frac{\left(\left(1+\frac{r}{n}\right)^{(nt)}-1\right)}{\left(\frac{r}{n}\right)}\right]$$

where FV is the future value, $pymt$ is the payment, r is the rate, t is the time in years and n is the number of compounding periods per year.

NOTE: The payment period and the compounding period will always match in our problems. Annuities which have the same payment and compounding period are called ***simple annuities***.

Example 8. Find the future value of an ordinary annuity with \$150 monthly payments at 6?% annual interest for 12 years.

Solution: For this problem we are given payment amount (\$150), the interest rate (0.0625 in decimal form), the compounding period (monthly or 12 periods per year), and finally the time (12 years). We plug each of these into the appropriate spot in the formula

$$FV = pymt\left[\frac{\left(\left(1+\frac{r}{n}\right)^{(nt)}-1\right)}{\left(\frac{r}{n}\right)}\right] = 150\left[\frac{\left(\left(1+\frac{0.0625}{12}\right)^{(12\cdot 12)}-1\right)}{\left(\frac{0.0625}{12}\right)}\right] = 32051.04651.$$

Since this is a monetary amount, we use two decimal places; using standard rounding the future value is \$32051.05.

A ***Christmas club account*** is a short-term special savings account usually set up at a bank or credit union in which a person can deposit regular payments for the purposes of saving money for Christmas purchases. Since there are regular payments, this is an ordinary annuity.

Example 9. On March 9, Mike joined a Christmas club. His bank will automatically deduct \$210 from his checking account at the end of each month, and deposit it into his Christmas club account, where it will earn $5\frac{1}{4}\%$ annual interest. The account comes to term on December 1. Find the following:

(1) the future value of Mike's Christmas club account;
(2) Mike's total contribution to the account;
(3) the total interest earned on the account.

Solution:

(1) For this part we will use the future value formula for an ordinary annuity. The payment amount is 210. The interest rate in decimal form is 0.0525. The number of compounding period in one year is 12 (monthly payments). The amount of time in years (t) is $\frac{9}{12}$. We will be making payments for 9 months (end of March, end of April, end of May, end of June, end of July, end of August, end of September, end of October, and finally end of November). This will give us

$$FV = 210\left[\frac{\left(\left(1+\frac{0.0525}{12}\right)^{\left(12\cdot\frac{9}{12}\right)}-1\right)}{\left(\frac{0.0525}{12}\right)}\right] = 1923.414866.$$

Mike's account will be worth \$1923.41 at maturity.

(2) To find the total amount of Mikes contribution, we only need to take the amount of each monthly payment and multiply by the number of payments per year and finally multiply by the number of years. This will give us $210 \cdot 12 \cdot \frac{9}{12} = 1890$. Thus the total contribution made by Mike is \$1890.

(3) To find the interest earned by Mike, we need to subtract his contribution from the future value. Hence $1923.41 - 1890 = 33.41$. Hence, Mike earned \$33.41 in interest on this account.

A ***tax-deferred annuity (TDA)*** is an annuity in which you do not pay taxes on the money deposited or on the interest earned until you start to withdraw the money from the annuity account.

Example 10. John Jones recently set up a tax-deferred annuity to save for his retirement. He arranged to have \$50 taken out of each of his biweekly checks; it will earn $8\frac{3}{8}\%$ annual interest. He just had his thirty-fifth birthday, and his ordinary annuity comes to term when he is sixty-five. Find the following:

(1) the future value of Johns annuity;
(2) Johns total contribution to the annuity;
(3) the total interest earned on the annuity.

Solution:

(1) For this part we will use the future value formula for an ordinary annuity. The payment amount is 50. The interest rate in decimal form is 0.08375. The number of compounding period in one year is 26 (bi-weekly payments every two weeks) . The amount of time in years (t) is calculated by taking the age at which Johns annuity comes to term and subtracting Johns current age (65-35 = 30). This will give us

$$FV = 50\left[\frac{\left(\left(1+\frac{0.08375}{26}\right)^{(26\cdot 30)}-1\right)}{\left(\frac{0.08375}{26}\right)}\right] = 175186.9942.$$

This will make the future value \$175,186.99.

(2) To find the total amount of Johns contribution, we only need to take the amount of each monthly payment and multiply by the number of payments per year and finally multiply by the number of years. This will give us $50 \cdot 26 \cdot 30 = 39000$. Thus the total contribution made by John is \$39,000.

(3) To find the interest earned by John, we need to subtract his contribution from the future value. We get $175185.99 - 39000 = 136185.99$. Thus John earns \$136,185.99 in interest on this account.

Example 11. Bill and Nancy want to set up a TDA that will generate sufficient interest at maturity to meet their living expenses, which they project to be \$1,500 per month.

(1) Find the amount needed at maturity to generate \$1500 per month interest if they can get $6\frac{3}{4}\%$ annual interest compounded monthly.

(2) Find the monthly payment they would have to put into an ordinary annuity to obtain the future value found in part a if their money earns $9\frac{1}{2}\%$ annual interest and the term is 30 years.

Solution:

(1) This first question is not an annuity problem at all. It is a basic simple interest problem, where we do not know the principal or the future value, but we do know the relationship between the two. We know that the amount of interest that needs to be earned in one month is 1500. This means that the future value will need to be the principal plus 1500 ($FV = P + 1500$). We are given r to be 0.06375, and t will be $\frac{1}{12}$ since this is only one month. The simple interest formula gives us

$$\begin{aligned}
1500 &= P \cdot 0.06375 \cdot \left(\frac{1}{12}\right) \\
1500 &= P(0.0053125) \\
\frac{1500}{0.0053125} &= P \\
P &= 282352.9412
\end{aligned}$$

This makes the needed principal (future value for out TDA) is \$282352.94 in order to obtain \$1500 in interest per month.

(2) For this part of the problem, we are being asked to find the monthly payments needed to end up with the future value of \$282,352.94 (see part (1)) in our annuity. We will use the future value of an ordinary annuity formula. We are given the future value of 282352.94. We have an interest rate for this annuity of 0.095 (different from part a), n is 12 (monthly payments and monthly compounding since it is a simple annuity), and t is given as 30 years. We need to solve for the payments. When we plug all this

into the formula we get

$$
\begin{aligned}
282352.94 &= pymt \left[\frac{\left(\left(1+\frac{0.095}{12}\right)^{(12\cdot 30)} - 1 \right)}{\left(\frac{0.095}{12}\right)} \right] \\
282352.94 &= pymt(2033.035174) \\
\frac{282352.94}{2033.035174} &= pymt \\
pymt &= 138.8824667.
\end{aligned}
$$

Normally in this class we use standard rounding, but in this case, we will want to round up to ensure that we end up with the desired future value in 30 years. Thus the monthly payments will need to be \$138.89.

4. Amortized Loan

A ***simple interest amortized loan*** is a loan whose principal is repaid over the life of the loan usually through equal payments. We usually refer to this simply as an ***amortized loan***. Meriam-webster.com give the following definition of amortizing: to pay off (as a mortgage) gradually usually by periodic payments of principal and interest.

Some examples of simple interest amortized loans are home loans (mortgages), car loans, and business loans

What we will be calculating with our loans will be the payment amount and total interest paid. In order to do these calculations will need the amount of the loan (principal), the interest rate, and the length of the loan. As with our simple annuities, the payment period and the compounding period will always be the same.

Amortized Loan Formula:

$$P\left(1+\frac{r}{n}\right)^{(nt)} = pymt \left[\frac{\left(\left(1+\frac{r}{n}\right)^{(nt)} - 1 \right)}{\left(\frac{r}{n}\right)} \right]$$

where P is the principal or loan amount, FV is the future value, $pymt$ is the payment, r is the rate, t is the time in years and n is the number of compounding periods per year.

Example 12. Find the monthly payment and total interest paid for a simple interest amortized loan of \$15,000 at an annual interest rate of $6\frac{3}{8}\%$ for 8 years.

Solution: For this problem we are given the laon amount (15000), the interest rate (0.06375 in decimal form), the compounding period (monthly or 12 periods per year), and finally the time (8 years). We plug each of these into the appropriate spot in the formula

$$\begin{aligned}
15000\left(1+\frac{0.06375}{12}\right)^{(12\cdot 8)} &= pymt\left[\frac{\left(\left(1+\frac{0.06375}{12}\right)^{(12\cdot 8)}-1\right)}{\left(\frac{0.06375}{12}\right)}\right] \\
24945.67081 &= pymt(124.808418) \\
\frac{24945.67081}{124.808418} &= pymt \\
pymt &= 199.8717011.
\end{aligned}$$

This will make the payment amount \$199.87.

Now we need to find the total interest paid. In this case the total amount paid will be more than the amount of the loan. From the first part we have the payment amount. So, we need to find the total amount paid and subtract the loan amount. As before, the total amount paid is the payment times the number of payments per year times the number of years. We get $199.87 \cdot 12 \cdot 8 = 19187.52$ for the total amount paid. Thus the interest paid is $19187.52 - 15000 = 4187.52$. The total interest paid on this loan is \$41878.52.

Example 13. Gloria bought a house for \$267,000. She put 20% down and obtained a simple interest amortized loan for the balance at $4\frac{7}{8}\%$ annual interest for 30 years.

(1) Find the amount of Glorias monthly payment.
(2) Find the total interest paid by Gloria.
(3) Most lenders will approve a home loan only if the total of all the borrowers monthly payments, including the home loan payment, is no more than 38% of the borrowers monthly income. How much must Gloria make in order to qualify for the loan?

Solution:

(1) Our first step here is to find the down payment on the house. To calculate the down payment we need to multiply the price of the house by 20%.

$$\text{down payment} = 267000 \cdot 0.20 = 53400$$

This means that Gloria will be paying \$53,400 in cash for the house and financing the rest with an amortized loan.

Now we need to find the amount of the loan. This will be the difference between the price of the house and the down payment.

$$267000 - 53400 = 213600$$

Now we are ready to use the amortized loan formula with the loan amount ($P = 213600$), the annual interest rate ($r = 0.04875$), and the number of years of the loan

($n = 30$). This will give us

$$
\begin{aligned}
213600\left(1+\frac{0.04875}{12}\right)^{(12\cdot 30)} &= pymt\left[\frac{\left(\left(1+\frac{0.04875}{12}\right)^{(12\cdot 30)}-1\right)}{\left(\frac{0.04875}{12}\right)}\right] \\
919327.5006 &= pymt(813.2843568) \\
\frac{919327.5006}{813.2843568} &= pymt \\
pymt &= 1130.388766
\end{aligned}
$$

This will make the monthly payment amount \$1130.39.

(2) To find the total interest paid by Gloria, we will use the same method as in example 12 with Glorias loan amount and the monthly payment that we just calculated. This will give us $I = \$1130.39 \cdot 12 \cdot 30 - \$213600 = \$193340.40$.

(3) To answer this question, we have to make some assumptions. The biggest assumption that we need to make is that Gloria has no other monthly expenses other than the monthly mortgage payment. We need the mortgage payment to be no more than 38% of Glorias monthly income. We can write this as an equation that looks like

$$pymt = (\text{monthly income}) \cdot 0.38.$$

We can solve this equation for monthly income to find out the minimum monthly income allowed for the payment.

$$
\begin{aligned}
1130.39 &= (\text{monthly income}) \cdot 0.38 \\
\frac{1130.39}{0.38} &= \text{monthly income} \\
\text{monthly income} &= 2974.710526.
\end{aligned}
$$

In this class we would normally use standard rounding, however, in order to ensure that the payment is no more than the monthly income, we will round up. Hence her monthly income should be at least \$2974.72.

You will find much more information on home loans and their cost by reading through the Home Loan Example. This also includes an amortization schedule. It is strongly recommended that you read through that entire example.