# Kinematics I 

Spring 2005

## What you should know

- What is the kinematics?
- Coordinate systems
- The difference between scalar and vector?
- Vector summation method (graphically and mathematically)
- Distance and displacement?
- Speed and Velocity?
- Acceleration?
- Tangential Method (Instantaneous vs. Average)


## What is the kinematics?

- Descriptions of Motion

Position, Velocity, and Acceleration

Linear Motion vs. Angular Motion
$\rightarrow$ Linear Kinematics and Angular Kinematics

## Linear Motion (=Translation)

- Rectilinear

- No change in both $\qquad$ of motion and $\qquad$ of the object during movement
- All points on the object move the same distance.

- Curvilinear
- Change in direction of motion, but No change in $\qquad$ of the object



## Angular Motion (=Rotation)

- Occur about an within a body or outside of the body.
- Continual change of orientation during movement.
- As the object moves, the paths that each point follows $\qquad$ path with having same center or axis


## General Motion (Translation+Rotation)

- Combination of linear and angular motions



## Descriptions

- Position : location in space
- Where is it now? $\rightarrow$ Need information.
system with reference point (origin)


## Descriptions

- : Describe the position with $\boldsymbol{r}$ (resultant) and $\theta$.

$$
X=r \cdot \cos \theta, \quad Y=r \cdot \sin \theta
$$

$(15,80)=\left[81.4 \cos \left(79.4^{\circ}\right), 81.4 \sin \left(79.4^{\circ}\right)\right]$

- Cartesian coordinate vs Polar coordinate


$$
\begin{aligned}
& \sin \theta=3 / r \rightarrow 3=r \sin \theta, \mathrm{r}=3 / \sin \theta \\
& \cos \theta=5 / r \rightarrow 5=r \cos \theta, \mathrm{r}=5 / \cos \theta
\end{aligned}
$$

## Scalar vs. Vector

- Scalar: physical quantity that is completely described by its
- No negative quantity ex) mass, time, distance, speed, ....
- Vector: physical quantity that possess both $\qquad$ and
ex) displacement, velocity, acceleration,....

Example: Walking north and east, then north and east again:


Overall distance traveled? P
Displacement ("as the crow flies")?
Magnitude d

Direction $\theta$

Examples - Angular distance and displacement.
Diving, gymnastics:
"triple somersault with a full twist"
3.0 revolutions about a transverse axis (somersaulting axis) combined with 1 rev about a longitudinal axis (twisting axis) transverse axis $\phi=\ldots \quad \theta=$ long. axis $\phi=\ldots \quad \theta=$

Discus throw:
Body rotates through ____ rotations prior to discus release.

$$
\phi=\ldots \quad \operatorname{rev} \theta=\ldots \quad \mathrm{rev}
$$

Joint range of motion:
The elbow and knee have ROM's of approximately 150 degrees.

$$
\phi=\_\quad \theta=
$$

$\qquad$
Typical units - 3 used commonly: revolutions, radians, degrees

$$
\begin{aligned}
& 1 \mathrm{rev}=2 \mathrm{~B} \text { rad (i.e., } 6.28 \mathrm{rad})=360 \mathrm{deg} \\
& 1 \mathrm{rad}=57.3 \mathrm{deg}
\end{aligned}
$$

## Distance Traveled and Displacement How far?

- Distance Traveled: a measure of the length followed by the object whose motion is being described, from its starting (initial) position to its ending (final) position.
- Scalar quantity (No direction)
- $\operatorname{Svmbol}=\lambda$

- Resultant Displacement: the length
from the initial position to the final position
- Vector quantity
- Symbol = d
- Calculation?
- Negative?



## $\not \subset$ How fast?

Describing rate of change of linear or angular position with respect to (writ) time

Speed or Velocity: Rate at which a body moves from one position to another

## Speed (scalar)

Velocity (vector)
Note: There will be times when it is important to use velocity because of its vector characteristic.

Linear:
Angular:

$$
\bar{v}=\frac{d}{\Delta t}
$$

$$
\bar{\omega}=\frac{\theta}{\Delta t}
$$

Examples of linear speed or velocity:
Tennis: $125 \mathrm{mph}(55.9 \mathrm{~m} / \mathrm{s})$ serve

$$
\mathrm{v}=125 \mathrm{mph}=55.9 \mathrm{~m} / \mathrm{s}
$$

Pitching: $90 \mathrm{mph}(40.2 \mathrm{~m} / \mathrm{s})$ fastball $\mathrm{v}=90 \mathrm{mph}=40.2 \mathrm{~m} / \mathrm{s}$

Running:
Marathon (26.2 mi) - 2 hr 10 min
(2.17 hr)

$$
\overline{\mathrm{v}}=12.1 \mathrm{mph}(5.4 \mathrm{~m} / \mathrm{s} ; 4: 57 \text { per mile })
$$

Sprinting - 100 m in 9.86 s

$$
\overline{\mathrm{v}}=10.14 \mathrm{~m} / \mathrm{s}=22.7 \mathrm{mph}
$$

Football - "4.6" speed ( 40 yd in 4.6 s )
$\overline{\mathrm{v}}=7.95 \mathrm{~m} / \mathrm{s}=17.8 \mathrm{mph}$
Typical units of measurement for v : $\mathrm{m} / \mathrm{s}, \mathrm{km} / \mathrm{hr}(\mathrm{kph})$, ft/s, mph

Examples of angular speed or velocity:
Auto engine speed - 3000 rpm
$\omega=3000 \mathrm{rpm}$
CD ROM drive - 400 rpm
$\omega=400 \mathrm{rpm}$
Cycling cadence - 90 rpm
$\omega=90 \mathrm{rpm}$
Isokinetic dynamometer (Cybex and others) e.g., 60 deg/s
$\omega=60 \mathrm{deg} / \mathrm{s}$

Body segment $\omega$ 's egg., peak $\omega$ of soccer player's knee... $2400 \mathrm{deg} / \mathrm{s} .6 .7 \mathrm{rev} / \mathrm{s}$

Typical units of measurement for $\omega$ : rpm, deg/s, rads

## Average vs. Instantaneous Velocities



Average speed per race distance: 100 m... marathon
Race that produces highest average speed?


Sometimes $\overline{\mathrm{V}}$ over relatively long t is not very informative - reflects need for a "kinematic profile" for more detailed information about performance

## e.g., distance running - split times

e.g., sprinting - 1987 T\&F World Champ. Johnson vs. Lewis
9.83 s
9.93 s
$\Delta=0.100 \mathrm{~s}$

Where was race won or lost?
Reaction time: $\quad \Delta=0.067 \mathrm{~s}$
After $10 \mathrm{~m}: \quad \Delta=0.100 \mathrm{~s}$
By decreasing tover which we examine velocity and other kinematic information, we approach instantaneous estimates of performance - gives more detail about performance.
e.g., during a soccer kick (next page):

## Ben vs. Carl

## Table $5.1 \& 5.2$ Result of 1988 Seoul Olympics

|  | Ben Johnson |  | Carl Lewis |  | Elapsed time diff(s) (Ben - Carl) | Interval time diff | Interval (m) | Ben <br> Avg. Speed <br> $(\mathrm{m} / \mathrm{s})$ | Carl <br> Avg. Speed <br> $(\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Elapsed time(s) | Interval time (s) | Elapsed time(s) | Interval time(s) |  |  |  |  |  |
| Position (m) |  |  |  |  |  |  |  |  |  |
| 0 | 0 |  | 0 |  |  |  |  |  |  |
| 10 | 1.83 | 1.83 | 1.89 | 1.89 | -0.06 | -0.06 | 0-10 | 5.46 | 5.29 |
| 20 | 2.87 | 1.04 | 2.96 | 1.07 | -0.09 | -0.03 | 10-20 | 9.62 | 9.35 |
| 30 | 3.8 | 0.93 | 3.9 | 0.94 | -0.1 | -0.01 | 20-30 | 10.75 | 10.64 |
| 40 | 4.66 | 0.86 | 4.79 | 0.89 | -0.13 | -0.03 | 30-40 | 11.63 | 11.24 |
| 50 | 5.5 | 0.84 | 5.65 | 0.86 | -0.15 | -0.02 | 40-50 | 11.9 | 11.63 |
| 60 | 6.33 | 0.83 | 6.48 | 0.83 | -0.15 | 0 | 50-60 | 12.05 | 12.05 |
| 70 | 7.17 | 0.84 | 7.33 | 0.85 | -0.16 | -0.01 | 60-70 | 11.9 | 11.76 |
| 80 | 8.02 | 0.85 | 8.18 | 0.85 | -0.16 | 0 | 70-80 | 11.76 | 11.76 |
| 90 | 8.89 | 0.87 | 9.04 | 0.86 | -0.15 | 0.01 | 80-90 | 11.49 | 11.63 |
| 100 | 9.79 | 0.9 | 9.92 | 0.88 | -0.13 | 0.02 | 90-100 | 11.11 | 11.36 |

## Ben vs. Carl

Ben vs Cark


Ben vs Carl


Interval (m)

$\not \subset$ Acceleration
Describing the rate of change of linear or angular velocity wrt time

Vector only - no scalar equivalent

Linear:
Angular:

$\square$

Typical units of measurement for acceleration:
Linear - $\mathrm{m} / \mathrm{s}^{2}$, $\mathrm{ft} / \mathrm{s}^{2}$
Angular - deg $/ \mathrm{s}^{2}, \mathrm{rad} / \mathrm{s}^{2}$
Example - linear
An athlete described as being "quick" or having "good acceleration" - same as being fast?

No, acceleration describes a person's ability to change speed or direction; doesn't describe top speed.

Example - angular
Throwing a baseball
Angular speed of shoulder internal rotation increases from zero to 1800 deg/s in 26 ms just prior to release...

Ball velocity at release correlates strongly ( $r=.75$ ) with shoulder internal rotation speed at release (Sherwood, 1995).

Example - linear:
Auto accelerations - Indy car vs. Corvette

| Indy car |  |  |  | Corvette |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta \mathrm{mph}$ | $\Delta \mathrm{m} / \mathrm{s}$ | $\Delta \mathrm{t}(\mathrm{s})$ | $\overline{\mathrm{a}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\Delta \mathrm{mph}$ | $\Delta \mathrm{m} / \mathrm{s}$ | $\Delta \mathrm{t}(\mathrm{s})$ | $\overline{\mathrm{a}}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |
| $0-60$ | $0-26.8$ | 2.0 | 13.42 | $0-30$ | $0-13.4$ | 1.8 | 7.45 |
| $60-80$ | $26-8-35.8$ | 0.9 | 9.94 | $30-40$ | $13.4-17.9$ | 1.0 | 4.47 |
| $80-100$ | $35.8-44.7$ | 1.1 | 8.13 | $40-50$ | $17.9-22.4$ | 1.5 | 2.98 |
| $100-120$ | $44.7-53.7$ | 1.2 | 7.45 | $50-60$ | $22.4-26.8$ | 1.7 | 2.63 |
| $120-140$ | $53.7-62.6$ | 1.4 | 6.39 | $60-70$ | $26.8-31.3$ | 1.8 | 2.48 |
| $140-160$ | $62.6-71.6$ | 1.5 | 5.96 | $70-80$ | $31.3-35.8$ | 2.2 | 2.03 |
| $160-180$ | $71.6-80.5$ | 1.5 | 5.96 | $80-90$ | $35.8-40.3$ | 2.8 | 1.60 |




When is speed or velocity highest for Indy car?

When is acceleration highest for Indy car?
Is the speed or velocity of the Indy car changing approximately 6-8 seconds after the start?

## Link between position, velocity, and acceleration:

Velocity - rate of change of position wrt time
Acceleration - rate of change of velocity wrt time X Instantaneous velocity is reflected by the slope of the position curve at some instant in time.

Instantaneous acceleration is reflected by the slope of the velocity curve at some instant in time.

See soccer kick - angular position, velocity, and acceleration graphs next page

## Tangential Method (Displacement $\rightarrow$ Velocity)

1. Find out " " (slope + to - or $-\mathbf{t o}+$ )

- Mark the instance (the specific time)
- Velocity $=0$ at the instance

2. Find out " " (slope $\uparrow \& \downarrow$ or $\uparrow \& \downarrow$ )

- Mark the instance (the specific time)
- $\quad$ Velocity = peak at the instance (flexion point of velocity)

3. Categorize positive and negative slope area

- d uphill $\rightarrow$ V positive, d downhill $\rightarrow \mathrm{V}$ negative
- Watch out deflexion point!!

4. Create the velocity curve based on the slope of displacement graph.



Return
Time (s)


Return
Time (s)


Return
Time (s)


| Six Cases of Acceleration |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | Change in <br> Speed | Starting <br> Direction | Ending <br> Direction | $\Delta \mathrm{v}$ | Accel. <br> $(\Delta \mathrm{v} / \Delta \mathrm{t})$ |  |
| 1 | Speeding <br> up |  |  |  |  |  |
| 2 | Slowing <br> down |  |  |  |  |  |
| 3 | Speeding <br> up |  |  |  |  |  |
| 5 | Slowing <br> down | Reversing <br> directions |  |  |  |  |
| 6 | Reversing <br> directions |  |  |  |  |  |

## Notes:

1. Speeding up in the positive direction $=$ $\qquad$ acceleration
2. Slowing down in the positive direction $=$ $\qquad$ acceleration
3. Speeding up in the negative direction $=$ $\qquad$ acceleration
4. Slowing down in the negative direction $=$ $\qquad$ acceleration
5. Reversing directions from positive to negative $=$ $\qquad$ acceleration
6. Reversing directions from negative to positive $=$ $\qquad$ acceleration.
