Linear and angular kinematics

- How far?
  - Describing change in linear or angular position
  - Distance (scalar): length of path
  - Displacement (vector): difference between starting and finishing positions; independent of path; “as the crow flies”
  - Symbols:
- linear - d   angular - θ

Examples of linear distance

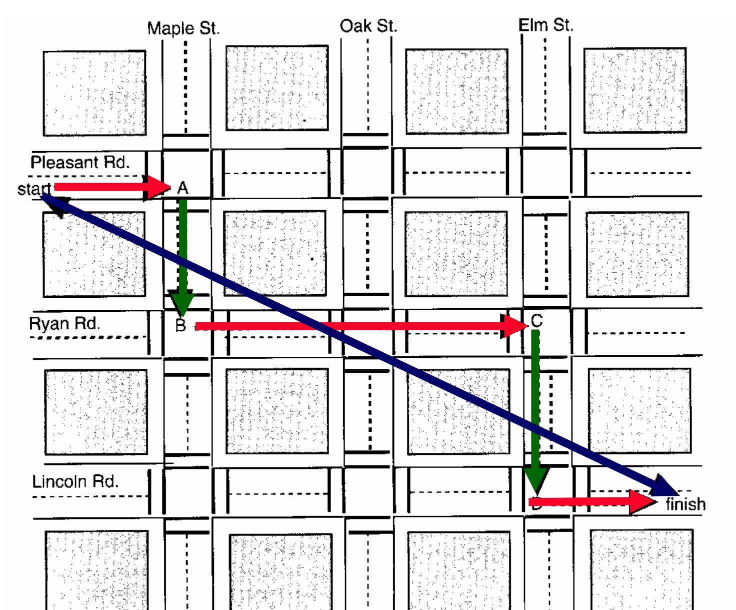
- Describing race distances:
  - 100 m sprint
  - Indy 500 auto race
  - 4000 km Tour de France
- Characterizing performance:
  - Shot put distance
  - Long jump distance
  - Pole vault height
- Typical units: cm, m, km, ft, mile

Stoichiometry example

- How far is 60 ft in m?
  - 1 ft = 12 in;
  - 1 in = 2.54 cm;
  - 100 cm = 1 m

$$60\,\cancel{ft} = \frac{12\,\cancel{in}}{1\,\cancel{ft}} \cdot \frac{2.54\,\cancel{cm}}{1\,\cancel{in}} \cdot \frac{1\,m}{100\,\cancel{cm}} = \frac{60 \cdot 12 \cdot 2.54 \cdot 1}{1 \cdot 1 \cdot 100}\,m = 18.288\,m$$

Difference between distance and displacement



Examples of angular distance

- Diving, gymnastics:
  - "two and a half with a full twist"
  - "triple toe loop"
- Typical units: three common units
  - Revolutions
  - Radians
  - Degrees

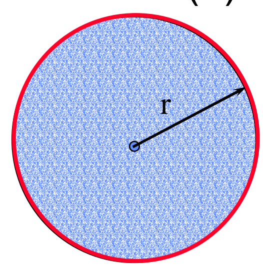
Stoichiometry example

- How many radians in 3 revolutions?
  - 1 rev = 2π rad (6.28 rad) = 360°
  - 1 rad = 57.3°

$$3\ rev = \frac{360\ deg}{1\ rev} \cdot \frac{2\pi\ rad}{360\ deg} \cdot = \frac{3 \cdot 360 \cdot 2\pi}{1 \cdot 360}\ rad = 18.84\ rad$$

What the heck is a radian?

- A radian is defined as the ratio between the circumference of a unit circle and the length of its radius (1):



- Circumference = $2\pi r$, so $C/r = 2\pi$; $C/1 = 2\pi$

Speed and velocity

– How Fast?
  - Describing the rate of change of linear or angular position with respect to time
  - Speed or velocity: Rate at which a body moves from one position to another
    – Speed (scalar)
    – Velocity (vector)
  - Linear: $\overline{v} = \frac{\Delta d}{\Delta t}$ Angular: $\overline{\omega} = \frac{\Delta \theta}{\Delta t}$

Examples of linear speed or velocity

- Tennis: 125 mph (56 m/s) serve
- Pitching: 90 mph (40 m/s) fastball
- Running:
  - Marathon: 26.2 mi in 2 hr 10 min
    - v = 12.1 mph = 5.4 m/s
  - Sprinting: 100 m in 9.80 s
    - v = 10.20 m/s = 22.95 mph
  - Football: "4.4 speed" (40 yd in 4.4 s)
    - v = 9.09 m/s = 20.45 mph
- Typical units: m/s, km/hr, ft/s, mph

Examples of angular speed/velocity

- Cycling cadence: 90 rpm
- Body joint angular velocities:
  - Kicking: soccer player's peak knee extension
    - ω = 2400 deg/s = 6.7 rev/s
  - Throwing: pitcher's peak elbow extension
    - ω = 1225 deg/s = 3.4 rev/s
  - Jumping: volleyball player's peak knee extension
    - ω = 974 deg/s = 2.7 rev/s
- Typical units: deg/s, rad/s, rpm

Acceleration

- Acceleration
  - Describes rate of change of linear and angular velocity with respect to time.
  - Vector only - no scalar equivalent
  - Linear: $\overline{a} = \frac{\Delta v}{\Delta t}$ Angular: $\overline{\alpha} = \frac{\Delta \omega}{\Delta t}$

- Example – angular acceleration
  - Throwing a baseball
    - Ball velocity correlates quite strongly (r = .75) with shoulder internal rotation speed at release (Sherwood, 1995).
    - Angular speed of shoulder internal rotation increases from zero to 1800 deg/s in 26 ms just prior to release...

$$\overline{\alpha} = \frac{(1800^\circ / s - 0^\circ / s)}{.026 s} = 69{,}230^\circ / s^2$$

  - Typical units:
    - Linear: m/s^2, ft/s^2
    - Angular: deg/s^2, rad/s^2

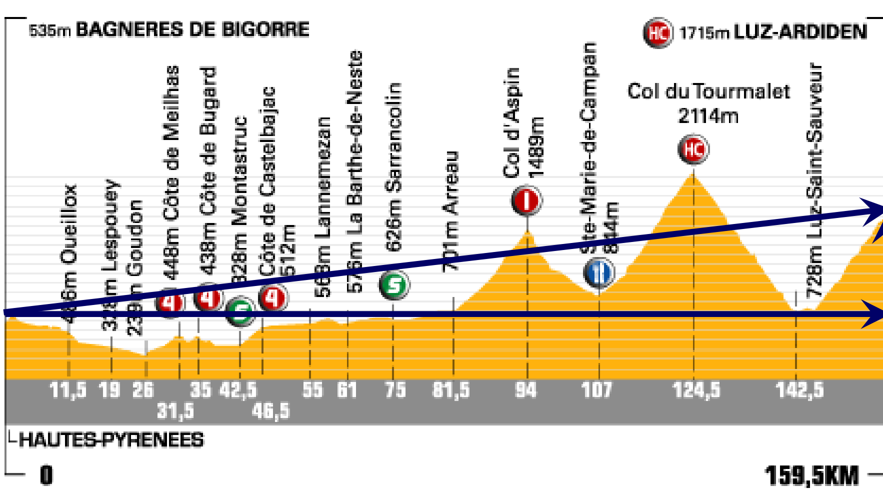
2003 Tour de France

Distance = 3427.5 km

Time = 83 h 41 m 12 s

Average speed =

Stage 15 kinematics



Instantaneous vs. average velocity

- **Average** velocity may not be meaningful in actions where many changes in direction occur.
- **Instantaneous** velocity is usually more important
  - specifies how fast and **in what direction** one is moving at one particular point in time
  - magnitude of instantaneous velocity is exactly the same as instantaneous speed

Instantaneous measures

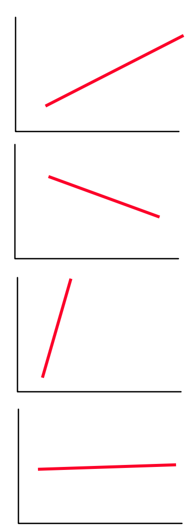
- Distance running: split times
  - Decreasing time over which we examine kinematic information gives us more detail about performance.
- Sprinting: 1987 T&F World Championship
  - Johnson (9.83 s) vs. Lewis (9.93 s)
  - Difference: Δt = 0.100 s. But, where was the race won or lost?

IMPORTANT

- Association between position, velocity, and acceleration:
  - Velocity: rate of change of position w.r.t. time
  - Acceleration: rate of change of velocity w.r.t. time
  - *Instantaneous velocity* is reflected by the slope of the position curve at some instant in time.
  - *Instantaneous acceleration* in reflected by the slope of the velocity curve at some instant in time.

Changes in a curve

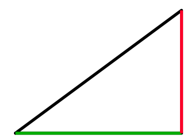
- positive change
  - up and to the right
- negative change
  - down and to the left
- quick change
  - very steep curve
- slow change
  - very flat curve



Slope of a Curve

- "Slope" = number which describes the change in a curve

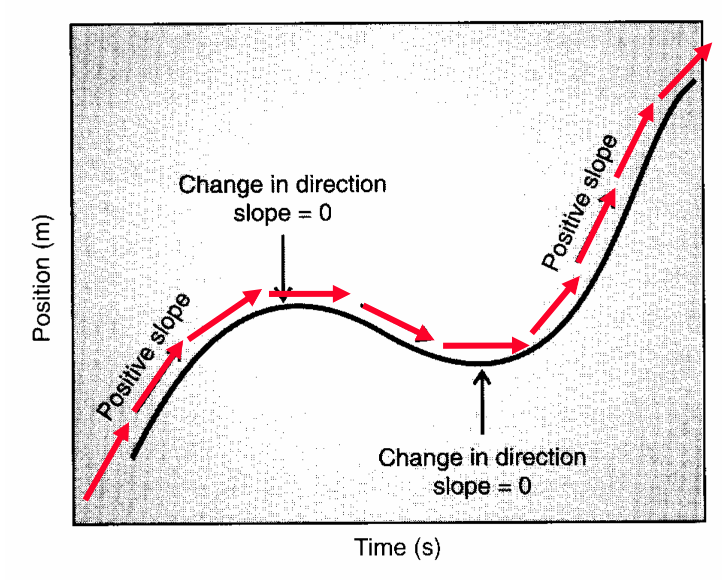
  – ***rise*/*run***



  – Note: this is the definition for the **tangent** of the lower angle in the triangle

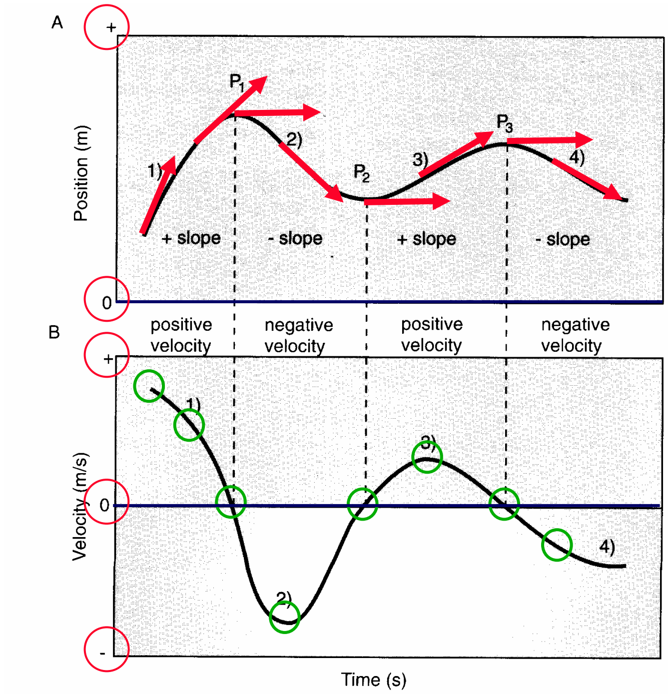
Tangent of a Curve

- tangent is drawn at only one pt on the curve
- a straight line which 'touches' the curve only at the one point
- slope of the tangent represents the slope of the curve
- Note: when person (object) changes direction the tangent is horizontal so the slope is ZERO



Relationship of *v* to *d*

- the instantaneous velocity (*v*) curve is the plot of how the slope of the *d* vs. *t* curve changes
- a similar relationship exists between *a* and *v*



Steps to determining *v* vs. *t* curve from *d* vs. *t* curve

(1) draw a set of axes (*v* & *t*) directly under the *d* vs. *t* curve

(2) locate all points where there is a change in direction

(3) plot zero velocity points for each corresponding change in direction

(4) between zero points identify if the slope of the curve is positive or negative

(5) determine how 'quickly' the slope changes

(6) estimate the shape of the *v* vs. *t* curve based on the direction and the steepness of the slope

SUMMARY: Displacement and Velocity

- Velocity = slope of displacement vs. time curve (slope = “rise”/”run”; $v = \Delta d/\Delta t$)
  - positive slope = positive velocity
  - negative slope = negative velocity
  - steeper slope = larger velocity
  - flatter slope = smaller velocity
  - no slope (horizontal) = 0 velocity
    - max or min position = 0 velocity
  - steepest slope = peak velocity

SUMMARY: Velocity and Acceleration

- Acceleration = slope of velocity vs. time curve (slope = “rise”/”run”; $a = \Delta v/\Delta t$)
  - positive slope = positive acceleration
  - negative slope = negative acceleration
  - steeper slope = larger acceleration
  - flatter slope = smaller acceleration
  - no slope (horizontal) = 0 acceleration
    - max or min velocity = 0 acceleration
  - steepest slope = peak acceleration