

Sets and Counting Review Solutions

1. If $n(U) = 150$, $n(A) = 37$, $n(B) = 84$, and $n(A \cup B) = 100$, find $n(A \cap B)$.

Solution:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$100 = 37 + 84 - n(A \cap B)$$

$$100 + n(A \cap B) = 121$$

$$n(A \cap B) = 121 - 100 = 21$$

2. How many cards in a standard deck of 52 cards are aces or spades?

Solution:

$$n(\text{aces or spades}) = n(\text{aces}) + n(\text{spades}) - n(\text{aces and spades})$$

$$n(\text{aces or spades}) = 4 + 13 - 1 = 16$$

3. A department store surveyed 428 shoppers and obtained the following information

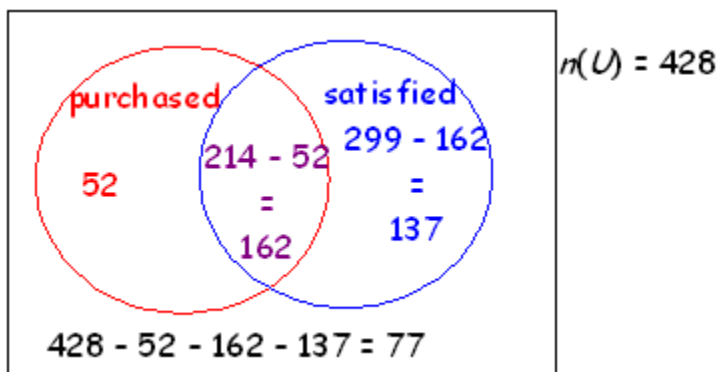
214 shoppers made a purchase.

299 shoppers were satisfied with the service.

52 of those shoppers who made a purchase were not satisfied with the service they received.

How many shoppers were satisfied with the service but did not make a purchase?

Solution:



The number of shoppers who were satisfied with the service but did not make a purchase are in the part of the satisfied circle that does not overlap with the purchased circle and is 137.

4. If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A = \{1, 2, 3, 4, 5\}$, and $B = \{4, 5, 6, 7, 8\}$, then find the set $n(A \cap B)$.

Solution:

This is the set of items that occur in both the set A and the set B .

$$n(A \cap B) = \{4, 5\}.$$

5. If you buy 3 pairs of jeans, 4 sweaters, and 2 pairs of boots, how many new outfits (each consisting of a pair of jeans, a sweater, and a pair of boots) will you have?

Solution:

This is a fundamental counting principle problem. We have 3 things to choose from: jeans, sweaters, and boots. We make a blank for each type of item, fill in the blank with the number of possible choices for that item and then multiply the numbers that we have together.

$$\underline{3} * \underline{4} * \underline{2} = 24.$$

6. From an English class consisting of 24 students, three students are to be chosen to give speeches in a school competition. In how many different ways can the teacher choose the 3 students if the order in which the students are selected is important?

Solution:

When order is important, we use permutations. In this case we are finding permutations of 24 things taken 3 at a time. ${}_{24}P_3 = 12144$

7. From an English class consisting of 24 students, three students are to be chosen to give speeches in a school competition. In how many different ways can the teacher choose the 3 students if the order in which the students are selected is not important?

Solution:

When order is not important, we use combinations. In this case we are finding combinations of 24 things taken 3 at a time. ${}_{24}C_3 = 2024$

8. A soccer league has eight teams. If every team must play every other team once in the first round of league play, how many games must be scheduled?

Solution:

Here we are picking pairs of teams to play each other. It does not matter which team is picked first and which team is picked second. All we need to know is how many different pairs (sets of 2) teams there are when there are 8 teams to choose from. Since order is not important and there is no replacement, this is a combination problem. Thus we would need to calculate ${}_8C_2 = 28$