## Logic Review Solutions

1. What is true concerning the validity of the argument below? (hint: Use a Venn diagram.)
2. All pesticides are harmful to the environment.
3. No fertilizer is a pesticide.

Therefore, no fertilizer is harmful to the environment.

## Solution:


2. What is true concerning the validity of the argument below? (hint: Use a Venn diagram.)

1. All roads lead to Rome.
2. Route 66 is a road.

Therefore, Route 66 leads to Rome.

## Solution:


3. What number is most likely to come next in the following sequence?

$$
2,3,5,7,11
$$

$\qquad$

## Solution:

Since this is a list of the first 5 prime numbers, the next most likely number is the sequence is 13 .
4. Use the statements

$$
p \text { : it is a square }
$$

$q$. it is a rectangle
how would you represent the sentence "All squares are rectangles" in symbolic form?

## Solution:

You first need to translate the sentence into a form that we know how to put in symbols to get "If it is a square, then it is a rectangle". Now we rewrite this in symbols to get $p \rightarrow q$.
5. Write the negation of the statement "Some computers are priced under \$100."

## Solution:

No computers are priced under $\$ 100$.
6. Under what conditions is the statement $p \rightarrow q$ true?

## Solution:

| $p$ | $q$ | $p \rightarrow q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

The statement is true when $p$ is true and $q$ is true.
The statement is true when $p$ is false and $q$ is true.
The statement is true when $p$ is false and $q$ is false.
7. Determine if the following is true: $\sim(p \wedge q) \equiv \sim p \wedge \sim q$.

Solution:

| $p$ | $q$ | $(p \wedge q)$ | $\sim(p \wedge q)$ |
| :---: | :---: | :---: | :---: |
| T | T | T | F |
| T | F | F | T |
| F | T | F | T |
| F | F | F | T |


| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

The statement is not true since the final columns of the truth tables do not match up.
8. Determine if the following is true: $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

Solution:

| $p$ | $q$ | $(p \vee q)$ | $\sim(p \vee q)$ |
| :---: | :---: | :---: | :---: |
| T | T | T | F |
| T | F | T | F |
| F | T | T | F |
| F | F | F | T |


| $p$ | $q$ | $\sim p$ | $\sim q$ | $\sim p \wedge \sim q$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | F | F | F |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

The statement is true since the final columns of the truth tables match exactly.
9. Rewrite the following sentence in "if . . ., then . . ." form: "I take public transportation only if it is raining."

Solution:
If I take public transportation, then it is raining.
10. Determine if the following are logically equivalent: $p \rightarrow q$ and $(p \rightarrow q) \wedge(q \rightarrow p)$.

Solution:

| $p$ | $q$ | $p \rightarrow q$ |
| :---: | :---: | :---: |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |


| $p$ | $q$ | $(p \rightarrow q)$ | $(q \rightarrow p)$ | $(p \rightarrow q) \wedge(q \rightarrow p)$ |
| :---: | :---: | :---: | :---: | :---: |
| T | T | T | T | T |
| T | F | F | T | F |
| F | T | T | F | F |
| F | F | T | T | T |

Not logically equivalent since the final columns in the truth table do not match exactly.
11. Under what conditions will ( $\sim p \vee q$ ) have a truth value of false?

Solution:

| $p$ | $q$ | $\sim p$ | $(\sim p \vee q)$ |
| :---: | :---: | :---: | :---: |
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

The statement if false then $p$ is true and $q$ is false.
12. Write the converse, inverse, and contrapositive of the following statement and indicate which statements are equivalent:

I walk to work if it is not raining.

## Solution:

The standard order for the original conditional is:
If it is not raining, then I walk to work.
The converse of the statement is:
If I walk to work, then it is not raining.
(switch the premise and conclusion)
The inverse of the statement is:
If it is raining, then I do not walk to work.
(leave the order the same but write the negation of the premise and the negation of the conclusion)
The contrapositive of the statement is:
If I do not work to work, then it is raining.
(switch and negate the premise and conclusion)
The converse and inverse are equivalent. The contrapositive and the original statements are equivalent.
13. What is true concerning the validity of the argument below? (hint: Use a Venn diagram.)

1. All police officers ride motorcycles.
2. Some women are police officers.

Therefore, some women ride motorcycles.
Solution:


Since it is impossible to have the women circle overlap the police officer circle and not overlap the ride motorcycle circle, the argument is valid.
14. What is true concerning the validity of the argument below? (hint: Use a Venn diagram.)

1. All police officers ride motorcycles.
2. Some women ride motorcycles.

Therefore, some women are police officers.

## Solution:



Since it is possible to have the women circle overlap the ride motorcycle circle and not overlap the police officer circle, the argument is invalid.

