1. Given 13 morks is 1 gloe, 5 gloes is 1 flit, 7 kits is one lonk, and 10 lonks is 1 gall. Convert 90 morks per kit to flits per gall.

Solution:

To solve this problem we will need to do some multiplication by a faction equal to 1. We can multiply by more than a single fraction that is equal to 1 at a time, but for this problem we will only multiply by a single fraction equal to 1 at a time. The number that we are converting is 90 morks per kit is really the fraction $\frac{90\text{ morks}}{1\text{ kit}}$.

I am going to start by converting the morks to flits. Since the morks are on the top of the fraction, we will need it to be on the bottom of the fraction that is equal to 1 in order for the morks to cancel. Since 13 morks equals 1 gloe, the fraction equal to 1 that we will use is $\frac{1\text{ gloe}}{13\text{ morks}}$. This will give us

$$\frac{90\text{ morks}}{1\text{ kit}} \times \frac{1\text{ gloe}}{13\text{ morks}} = \frac{90\text{ gloes}}{13\text{ kits}}.$$

We now have to get from gloes to flits. Since there are 5 gloes in 1 flit, the fraction equal to 1 that we will use is $\frac{1\text{ flit}}{5\text{ gloes}}$. Since the gloes are in the denominator of the fraction, they will cancel with the gloes in our current fraction.

$$\frac{90\text{ gloes}}{13\text{ kits}} \times \frac{1\text{ flit}}{5\text{ gloes}} = \frac{90\text{ flits}}{65\text{ kits}}.$$

Now that we have the morks converted to flits, we need to convert the kits to galls. Since the kits are in the denominator of our fraction we will need a fraction equal to 1 that has kits in the numerator. This fraction will be $\frac{7\text{ kits}}{1\text{ lonk}}$. This will give us

$$\frac{90 \text{ flits}}{65 \text{ kits}} \times \frac{7 \text{ kits}}{1 \text{ lonk}} = \frac{630 \text{ flits}}{65 \text{ lonks}}.$$

We are almost done. We need to complete the solution by multiplying by the fraction $\frac{1 \text{ gall}}{10 \text{ lonks}}$ which is equal to 1. This will give us

$$\frac{630 \text{ flits}}{65 \text{ lonks}} \times \frac{10 \text{ lonks}}{1 \text{ galls}} = \frac{6300 \text{ flits}}{65 \text{ galls}}.$$

This can be simplified to $\frac{1260 \text{ flits}}{13 \text{ galls}}$ or $\frac{1260}{13}$ flits per gall.

2. Given that one ork is equivalent to 5 umphs, convert 2.14 square orks to square umphs.

Solution:

First we need to realize that square orks are $\text{orks} \times \text{orks}$. This means that we will have to convert each ork to umphs in order to get $\text{umphs} \times \text{umphs}$ which are square umphs.

As is normal with conversion, we will need to multiply by a fraction that is equal to 1. The number that we are converting is $\frac{2.14 \text{ square orks}}{1} = \frac{2.14 \text{ orks} \times \text{orks}}{1}$. Since orks are in the numerator, we will need our fraction that is equal to 1 to have orks in the denominator so that the orks will cancel. This means we will need to multiply by $\frac{5 \text{umphs}}{1 \text{ ork}}$. This will give us

$$\frac{2.14 \text{ orks} \times \text{orks}}{1} \times \frac{5 \text{ umphs}}{1 \text{ ork}} \times \frac{5 \text{ umphs}}{1 \text{ ork}} = \frac{53.5 \text{ umphs} \times \text{umphs}}{1}.$$

So 2.14 square orks is equal to 53.5 square umphs.

3. Given a brunk is equivalent to 5 plops, and a plop is equivalent to 4 nerd, convert 20 nerds to brunks.

Solution:

Our problem here is to convert 20 nerds which is the same as $\frac{20\text{ nerds}}{1}$ to brunks. This will take multiplying by two fractions that are equal to 1 since we do not have a single equivalency for nerds and brunks. We will have to go through plops first. Since the nerds are in the numerator, we will need a fraction equal to 1 with nerds in the denominator so that the nerds will cancel. This fraction is $\frac{1\text{ plops}}{4\text{ nerds}}$.

This will give us

$$\frac{20\text{ nerds}}{1} \times \frac{1\text{ plops}}{4\text{ nerds}} = \frac{20\text{ plops}}{4} = \frac{5\text{ plops}}{1}.$$

Now we have plops and need to finally make it to brunks. The fraction equal to 1 that we need to multiply by is $\frac{1\text{ brunk}}{5\text{ plops}}$. This fraction will allow us to cancel the plops since they are in the numerator of our problem and in the denominator of this fraction. This will give us

$$\frac{5\text{ plops}}{1} \times \frac{1\text{ brunk}}{5\text{ plops}} = \frac{5\text{ brunks}}{5} = 1\text{ brunk}.$$

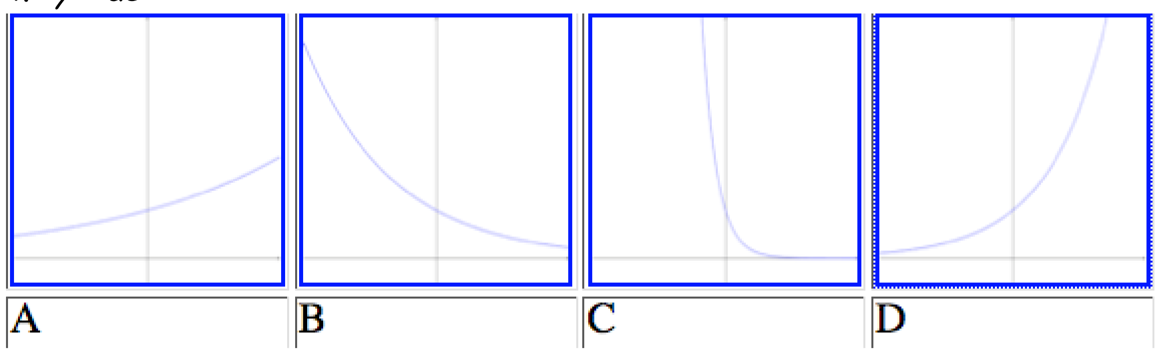
4. a. Use a calculator to estimate $5e^{0.2204*2}$ with four decimal place accuracy.
   b. Use a calculator to estimate ln(7) with four decimal place accuracy.

Solution:

a. How to do this calculation will be different depending on the calculator that you have. You should note that if you are doing the computation one piece at a time, you must first calculate the exponent 0.2204*2. Then you raise e to that answer that you got for the exponent. Finally, you would multiply that answer by 5. You should get 7.769749406. Four decimal place accuracy will give us 7.7697

b. How to do this calculation can differ depending on your calculator. The answer should be 1.945910149. Four decimal place accuracy will give us 1.9459

5. Match the equations 1 through 4 to the graphs A through D below:
   1. $y = ae^{0.25t}$
   2. $y = ae^{0.75t}$
   3. $y = ae^{-0.5t}$
   4. $y = ae^{-3t}$



Solution:

The only difference among the equations is the k part of the exponent ($y = ae^{kt}$). You should remember that a negative value of k will produce a graph that goes down from the left side of the graph to the right side of the graph. The other thing that we need to remember is that the bigger the absolute value of k, the faster the graph either goes up or goes down.

The equation $y = ae^{0.25t}$ has a k of 0.25. This tells us that the graph must go up as we look from left to right. The absolute value of .025 is .025. This a smaller absolute value than the other equation with a positive k, thus this graph must go up more slowly from left to right. Thus the equation $y = ae^{0.25t}$ would be matched with graph A.

The equation $y = ae^{0.75t}$ has a k of 0.75. This tells us that the graph must go up as we look from left to right. The absolute value of .075

is .075. This a larger absolute value than the other equation with a positive k, thus this graph must go up more quickly from left to right. Thus the equation $y = ae^{0.75t}$ would be matched with graph D.

The equation $y = ae^{-0.5t}$ has a k of -0.5. This tells us that the graph must go down as we look from left to right. The absolute value of -0.5 is 0.5. The is a larger absolute value than the other equation with a negative k, thus this graph must go down more slowly from left to right. Thus the equation $y = ae^{-0.5t}$ would be matched with graph B.

The equation $y = ae^{-3t}$ has a k of -3. This tells us that the graph must go down as we look from left to right. The absolute value of -3 is 3. The is a larger absolute value than the other equation with a negative k, thus this graph must go down more quickly from left to right. Thus the equation $y = ae^{-3t}$ would be matched with graph C.

6. Bacteria X has a relative growth rate of 230% under ideal conditions. Some bacteria X are accidentally introduced into some potato salad. Two hours after contamination, there were 24000 bacterial X in the potato salad.
   a. Find the initial number of bacteria X introduced into the potato salad.
   b. Estimate the number of bacteria in the food 3 hours after contamination.

Solution:

a. For this problem we need to figure out what each number is for us. Our basic equation is $y = ae^{kt}$ where a is the initial number of bacteria, k is the relative growth rate in decimal form, t is the amount of time, and y is the number of bacteria of the amount of time t.

   We are told that the relative growth rate is 230%. This would mean that k is 2.3. We are also told that after 2 hours, there are 24000 bacteria. This means that for this part of our problem t is 2 and y is 24000. We are asked to find a. When we plug everything into the equation we get $24000 = ae^{(2.3*2)}$. To solve for a in this equation, we just need to divide both sides of the equation by $e^{(2.3*2)}$ which is just a number. Thus we get

$$24000 = ae^{(2.3*2)}$$
$$\frac{24000}{e^{(2.3*2)}} = \frac{ae^{(2.3*2)}}{e^{(2.3*2)}}$$
$$\frac{24000}{e^{(2.3*2)}} = a$$
$$241.2440579 = a$$

Since a is a number of bacteria, it must be a whole number. Thus the initial number of bacteria is either 241 or 242 (either would be considered correct).

b. For this part of the question, we need to use our initial number of bacteria from part a (we will use 241), the growth rate given in the problem and the new time t equals 3. When we plug this into our basic equation $y = ae^{kt}$, we get

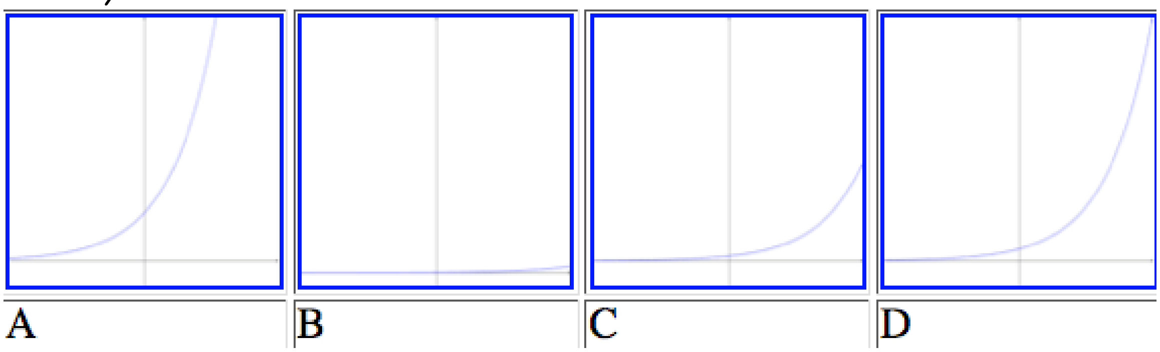
$$y = ae^{kt}$$
$$y = 241e^{(2.3*3)}$$
$$y = 239138.2065$$

Again since y is supposed to be a number of bacteria, we must round to a whole number. Thus either 239138 bacteria or 239139 bacteria could be considered correct.

Note: If you chose to use 242 for a, then you should get 240130 bacteria or 240131 bacteria.

7. Match the equations 1 through 4 to the graphs A through D below:

1. $y = \frac{1}{4}e^{kt}$
2. $y = 2e^{kt}$
3. $y = 20e^{kt}$
4. $y = 5e^{kt}$



Solution:

The only difference among the equations is the *a* part of the exponent ($y = ae^{kt}$). You should remember that the *a* part of the equation determines where the graph intersects the *y*-axis. The point of intersection will always be (0, *a*).

The equation $y = \frac{1}{4}e^{kt}$ has an *a* of $\frac{1}{4}$. This tells us that the graph will intersect the *y*-axis at the point $\left(0, \frac{1}{4}\right)$. This is the smallest value of *a*. Thus the equation $y = \frac{1}{4}e^{kt}$ would be matched with graph B.

The equation $y = 2e^{kt}$ has an *a* of 2. This tells us that the graph will intersect the *y*-axis at the point $(0, 2)$. This is the second smallest value of *a*. Thus the equation $y = 2e^{kt}$ would be matched with graph C.

The equation $y = 20e^{kt}$ has an *a* of 20. This tells us that the graph will intersect the *y*-axis at the point $(0, 20)$. This is the second largest value of a. Thus the equation $y = 20e^{kt}$ would be matched with graph A.

The equation $y = 5e^{kt}$ has an a of 5. This tells us that the graph will intersect the y-axis at the point $(0,5)$. This is the second smallest value of a. Thus the equation $y = 5e^{kt}$ would be matched with graph D.

8. Write the inverse of the exponential function $n(t) = 17800e^{0.09t}$.

Solution:

This function $n(t) = 17800e^{0.09t}$ is our basic exponential function $y = ae^{kt}$. We know that the inverse of our basic function is $t = \frac{1}{k}\ln\left(\frac{y}{a}\right)$. In the inverse, we will need to fill in numbers for the letters k and a. From the original problem, we see that a is 17800 and that k is 0.09. This means that the inverse function will be $t = \frac{1}{0.09}\ln\left(\frac{y}{17800}\right)$.

9. A bacteria culture initially contains 2000 bacteria and doubles every half hour. The formula for the population is $p(t) = 2000e^{kt}$ for some constant k.
   a. Find k for this bacteria culture.
   b. Find the size of the bacterial population after 20 minutes.
   c. Find the size of the bacterial population after 7 hours.

Solution:

a. To find k, we will need to fill in numbers for $p(t)$ (which is y in our basic equation) and for t. We are told that in $\frac{1}{2}$ hour, there will be twice as many bacteria as we started with (doubles every half hour). We know from the equation that a is 2000 and a is the initial or starting number of bacteria. This means that when t is $\frac{1}{2}$, there will be 2000*2 or 4000 bacteria. Thus 4000 will be our y for this part of the problem. When we plug this into the basic equation $y = ae^{kt}$, we get $4000 = 2000e^{k*0.5}$. In order to find k, we will need to use the inverse function of our exponential function ($t = \frac{1}{k}\ln\left(\frac{y}{a}\right)$). We know that y is 4000, a is 2000, and t is 0.5. When we plug all this into the inverse function, we get

$$0.5 = \frac{1}{k}\ln\left(\frac{4000}{2000}\right)$$
$$0.5 = \frac{1}{k}\ln(2)$$

We now need to solve this equation for k.

$$0.5 = \frac{1}{k}\ln(2)$$
$$0.5k = \ln(2)$$
$$k = \frac{\ln(2)}{0.5}$$
$$k = 1.386294361$$

b. For this part we will need to use our k from part a to find the number of bacteria after 20 minutes. Since we used 0.5 to represent ½ hour, we need to convert 20 minutes to hours. Using our dimensional analysis, we find that 20 minutes is

$$\frac{20\text{ minutes}}{1} \times \frac{1\text{ hour}}{60\text{ minutes}} = \frac{20\text{ hours}}{60} = \frac{1}{3}\text{ hours}$$

We now plug into our equation

$$y = p(t) = 2000e^{\left(1.386294361*\frac{1}{3}\right)}.$$
$$y = 3174.802104$$

Thus after 20 minutes there will be either 3174 bacteria or 3175 bacteria.

c. This part of the problem will work the same way as part b. We will just plug 7 in for t in the equation to get

$$y = p(t) = 2000e^{(1.386294361*7)}.$$
$$y = 32768000$$

Thus there will be 32768000 bacteria in 7 hours.

10. The number of bacteria in a culture is given by the function $n(t) = 975e^{0.4t}$ where t is measured in hours.
    a. What is the relative growth rate of this bacterium population?
    b. What is the initial population of the culture?
    c. How many bacteria will the culture contain at time $t = 5$?

Solution:

a. The relative growth rate in decimal form is given as k in the exponent of the basic function $y = ae^{kt}$. Thus the decimal form of the relative growth rate is 0.4. To change this to a percent, we multiply by 100. This will give us a relative growth rate of 0.4*100=40. Thus the relative growth rate is 40%.

b. Recall from the basic function $y = ae^{kt}$ that a is the initial population. Thus for our culture, the initial population is 975 bacteria.

c. To find out how many bacteria there will be in 5 hours, we only need to replace t in the equation with 5 and use our calculator. This will give us

$$n(t) = 975e^{(0.4*5)}$$
$$n(t) = 7204.329696$$

Thus in 5 hours there will be either 7204 bacteria or 7205 bacteria.

11. At the beginning of an experiment, a scientist has 148 grams of radioactive goo. After 150 minutes, her sample has decayed to 4.625 grams.
    a. What is the half-life of goo in minutes?
    b. Find a formula for $G(t)$, the amount of goo remaining at time t.
    c. How many grams of goo will remain after 62 minutes?

    Solution:

    a. To find the half-life of goo, we need to find how long (in minutes) it will take for us to end up with half of the goo that we start with. This would be a t. In order to find t we will need to have a (the initial amount of goo), $G(t)$ (the ending amount of goo), and k (the relative decay rate of goo). Although we have a and $G(t)$ (also know as y), we do not have k. We will have to calculate this first.

       In order to calculate k, we will need A and the t (time) and $G(t)$ (ending amount) that go together. We do have this. a is 148. T is 150 minutes for a $G(t)$ (also know as y) of 4.625. Since we are looking for k, we will use the formula $t=\frac{1}{k}\ln\left(\frac{y}{a}\right)$. When we plug this in, we get

$$
\begin{aligned}
t &= \frac{1}{k}\ln\left(\frac{y}{a}\right)\\
150 &= \frac{1}{k}\ln\left(\frac{4.625}{148}\right)\\
150k &= \ln\left(\frac{4.625}{148}\right)\\
k &= \frac{1}{150}\ln\left(\frac{4.625}{148}\right)\\
k &\approx -.023104906
\end{aligned}
$$

       Now that we have an approximation of k, we can calculate the half-life of goo. Since we are looking for t, we will use $t=\frac{1}{k}\ln\left(\frac{y}{a}\right)$. We need to fill in for a (148), k (-.02310496) and y. Since we are looking for the half-life, we need y to be half of the amount of goo we start with $\left(\frac{148}{2}=74\right)$. Once we plug everything in, we get

$$t = \frac{1}{k}\ln\left(\frac{y}{a}\right)$$
$$t = \frac{1}{-.023104906}\ln\left(\frac{74}{148}\right)$$
$$t = 30$$

Thus the half-life of the goo is 30 minutes.

b. Finding a formula for the amount of goo remaining after t minutes just means to plug a, and k into the basic exponential formula $y = G(t) = ae^{kt}$. This will give us $y = G(t) = 148e^{-.023104906\,t}$.

c. To find the amount of goo remaining after 62 minutes, we only need to plug 62 in for t in the formula that we found in part b. This will give us

$$y = 148e^{-.023104906 * 62}$$
$$y \approx 35.32913934$$

Thus there will be approximately 35.32914 grams of goo after 62 minutes.