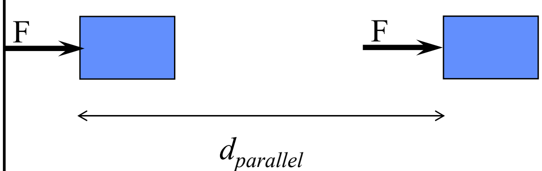
Work

Work = **Force** x "parallel distance" (parallel component of displacement)

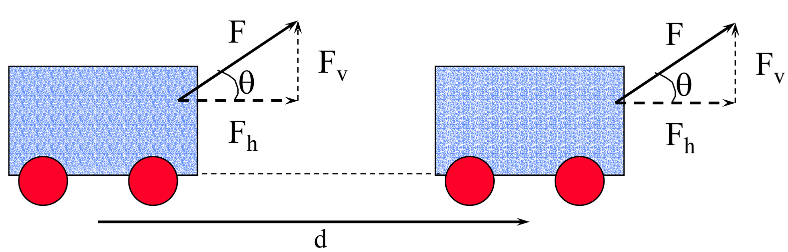


$$W_k = \overline{F} \cdot d_{parallel}$$

$\overline{F} =$ average force computed over the distance

$$Units: N \cdot m = J = \text{"joules"} = (kg \cdot m^2 / s^2)$$

When $\vec{F}$ is *not parallel* to $\vec{d}$, then we must take the *component* of F which is *parallel* to $\vec{d}$.



$$F_h = F\cos\theta$$
$$F_v = F\sin\theta$$
$$W_k = F_h d = F(\cos\theta)d$$

If F = 100 N and θ = 30 degrees,
Compute Fv and Fh
If d = 5 m, compute W

* for those of you who have had advanced math, the parallel component is computed by the *dot product* of the force and displacement vectors.

Example:
If you pull on a wagon with a force of 100 N at an angle of 30 degrees w.r.t. the horizontal and you pull over a distance of 5 m, how much work do you do on the wagon?

Know: $F = 100$ N; $d = 5$ m; $\theta = 30°$

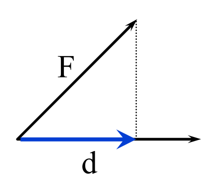
Need: W_k

Use: $W_k = (F \cdot d)_{parallel} = F \cdot \cos\theta \cdot d$

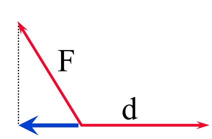
$W_k = 100\ \text{N} \cdot \cos 30° \cdot 5\ \text{m} = 433\ \text{N} \cdot \text{m}$

Positive v. Negative Work

If F and $d_{parallel}$ point in the **same** direction
the work done is <u>positive</u>



If F and $d_{parallel}$ point in **opposite** directions
the work done is <u>negative</u>



Energy = the capacity to do work (scalar)

Types of energy: mechanical, chemical, heat, sound, light, etc

We are most interested in mechanical energy

Mechanical Energy

- Kinetic Energy (KE) - energy due to **motion**

$$KE = \frac{1}{2}mv^2 \qquad unit: \ kg \cdot m^2 / s^2 = J$$

e.g. a diver (mass = 70 kg) hits the water after a dive from the 10 m tower with a velocity of 14 m/s. How much KE does she possess?

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}70\ kg(-14\ m/s)^2 = 6860\ J$$

Mechanical Energy

- (Gravitational) Potential Energy (P.E.)
  - energy due to the **change of position** in gravitational field
  - PE = –mgh

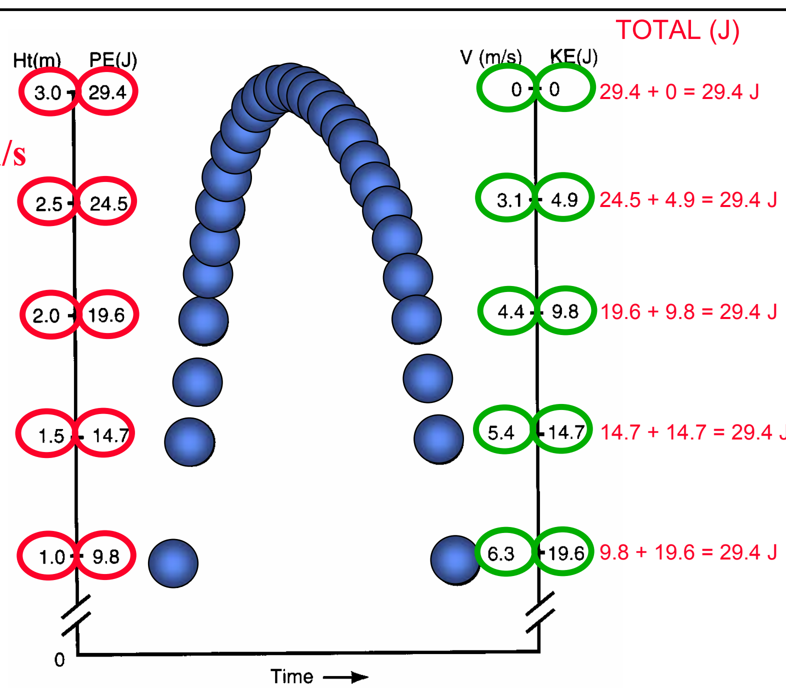
    h = height of something above some reference line

    m = **mass**

    g = **acceleration due to gravity (–9.8 m/s^2)**

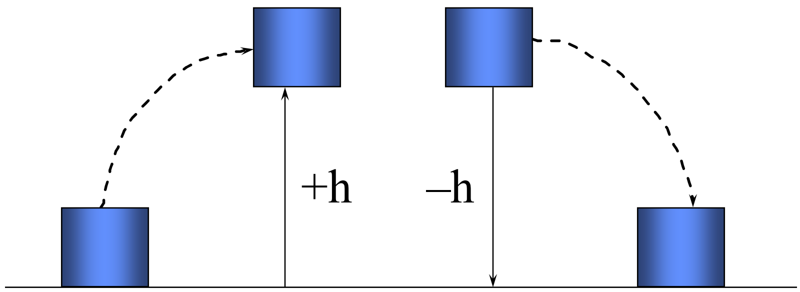
m = 1 kg

v_i = 6.3 m/s



$\Delta PE > 0$
$\Delta PE = -m(-g)(+h)$
$\Delta PE = +mgh$

$\Delta PE < 0$
$\Delta PE = -m(-g)(-h)$
$\Delta PE = -mgh$



$$Units: kg\cdot m^2/s^2 = J$$

The negative sign in the PE equation is necessary to account for the direction of gravity ($PE > 0$ when $h > 0$ and $PE < 0$ when $h < 0$)

Potential Energy:

Note: in the absence of air resistance and other resistive forces-- PE can be completely converted to KE by the work done by gravity on the way down.

e.g. a diver on top of a 10 m tower has a positive ΔPE compared to water level

$$\Delta PE = -mgh = -60\ kg\cdot(-9.8\ m/s^2)\cdot(+10\ m) = 6860\ J$$

NOTE : This value is identical to that found in the kinetic energy example.

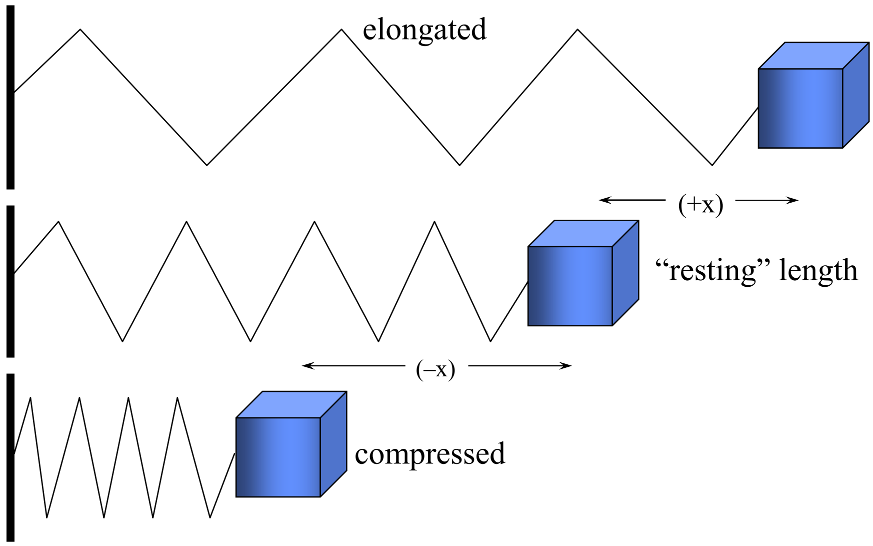
Mechanical Energy

- Strain or elastic energy (SE)
  - energy due to **deformation**
  - this type of energy arises in compressed springs, squashed balls ready to rebound, stretched tendons inside the body, and other deformable structures

$$SE = \frac{1}{2}kx^2$$

$$x = amount\ of\ deformation$$

$$k = stiffness\ of\ spring\ (or\ other\ structure)$$



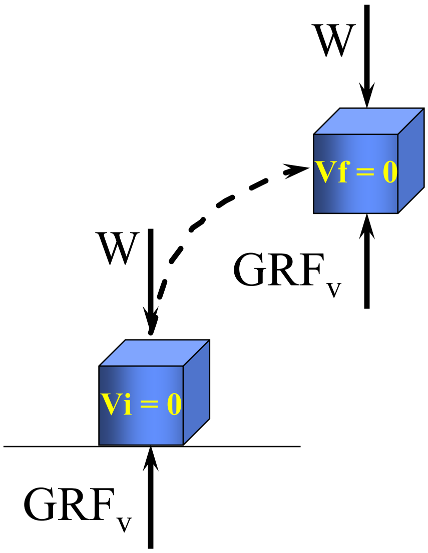
Work-Energy Relationship

The work done by the net force acting on a body is equal to the change in the body's **kinetic energy**

$$(\Sigma \overline{F} \cdot d_{parallel}) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\Sigma W_k = \Delta KE$$

This relationship is true as long as there is no change in vertical position.

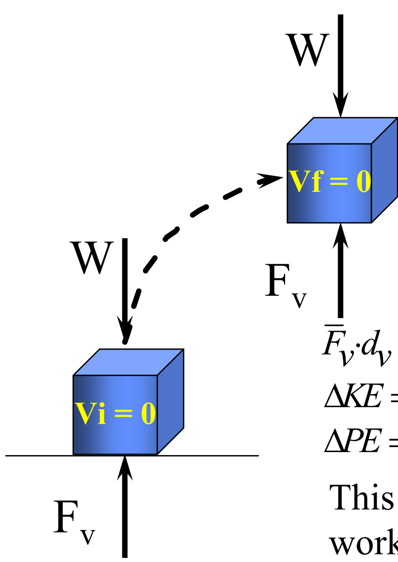


if $KE_i = 0$ and $KE_f = 0$ then $\Delta KE = 0$; therefore the work done by the person (GRF_v) is completely offset by the negative work done by gravity (W).

Overall -- no work was done because there was no change in KE overall

Tell that to your muscles!!!

Alternate formulation of Work-Energy Relationship (Hay, 1993)



Consider the **work done by a single force.** In this example consider F_v

$$\bar{F}_v \cdot d_v = \Delta KE + \Delta PE\,(+\Delta SE)$$

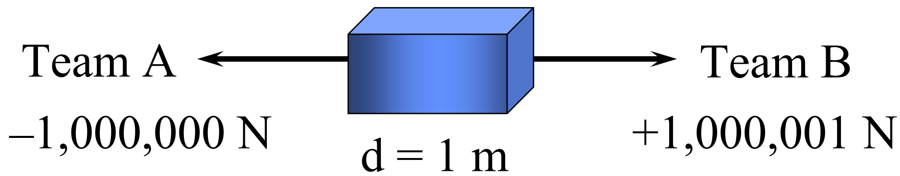
$$\Delta KE = 0$$

$$\Delta PE = -mgh = -100\,kg\,(-9.8\,m/s^2)\,2\,m = 1960\,J$$

This is numerically opposite to the work done by gravity.

F_v did +1960 J of work
W did –1960 J of work

Another example:



$$\Sigma F = -1{,}000{,}000\ \text{N} + 1{,}000{,}001\ \text{N} = 1\text{N}$$

$$W_k = \Sigma F \cdot d = 1\text{N} \cdot 1\text{m} = 1\text{Nm}$$

Example problems: Work

- A 60 diver leaves the 10 m board with zero vertical velocity.
  - Compute the potential and kinetic energies at 10 m, 5 m and 0 m above the water.
  - Compute the work done on the diver upon entry.
- A baseball catcher "gives" to stop a 22 m/s fastball over a distance of 0.25 m.
  - What was the average force needed to stop the 0.2 kg baseball?
  - What is the average force needed to stop the same 22 m/s fastball over 0.125 m?

Example problems: Work

- A 60 kg diver leaves the 10 m board with zero vertical velocity.
  - **Know**:
  - **Need**:
  - **Use**:

Example problems: Work

Distance (m)	PE (J)	KE (J)	Total Energy
10			
5			
0			
At rest			

Work done =

Example problems: Work

- A baseball catcher "gives" to stop a 22 m/s fastball over a distance of 0.25 m.
  - **Know**:
  - **Need**:
  - **Use**:

Power

- the rate at which work is done
- the rate at which energy is expended

$$\overline{P} = \frac{W_k}{t} = \frac{work}{time}$$

$$units: \frac{J}{s} = \frac{joule}{s} = watt\,(W)$$

Example

if m = 100 kg   g = 9.8 m/s²   h = 2 m

W_k = mgh = **100(9.8)(2) = 1960 J**

now add time

Case 1: raise the barbell slowly -- t = 5 s

$$\overline{P} = \frac{W_k}{t} =$$ **1960 J / 5 s = 392 W**

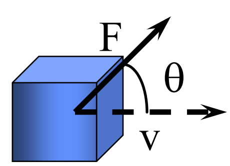
Case 2: raise the barbell quickly -- t = 1.5 s

$$\overline{P} = \frac{W_k}{t} =$$ **1960 J / 1.5 s = 1306.7 W**

Power - alternate form of equation

$$\overline{P} = \frac{\overline{W}_k}{t} = \frac{\overline{F} \cdot d_{parallel}}{t} = \overline{F} \cdot v_{parallel}$$

Power can be **positive or negative** depending on whether F and v point in same general direction (+ power) or in opposite directions (–power)



$$\overline{P} = F \cdot \cos\theta \cdot v$$

Example problems: Work & Power

•Arizona State University football coach Bruce Snyder banished 242 lb (1078 N) Terry Battle and 205 lb (913 N) Jake "The Snake" Plummer to the Sun Devil Stadium steps for some "corrective therapy" following a poor practice. The 50 stadium steps require a vertical rise of 15 m. Battle completed 8 trips up the stairs in 195 s and Plummer completed 12 trips in 270 s.

- –Which athlete performed more work?
- –Which athlete had the higher average power output?

(Ignore the return trips down the steps and any horizontal displacement)

Example problems: Work & Power

•Two soccer players work out in the off season by sprinting up a 40º hill for a distance of 100 m before stopping, resting, and walking back down. Mia has a mass of 60 kg. Julie has a mass of 65 kg. It takes Mia 42 sec. and Julie 38 sec. to reach the top of the hill.

- –Which athlete performed more work?
- –Which athlete had the higher average power output?
- (Ignore the return trips down the steps and any horizontal displacement)