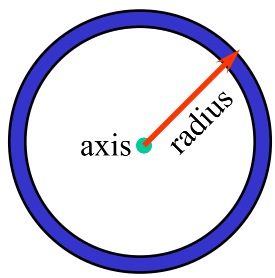
Relationships between linear and angular motion

- Body segment rotations combine to produce linear motion of the whole body or of a specific point on a body segment or implement
  - Joint rotations create forces on the pedals.
  - Forces on pedals rotate crank which rotates gears which rotate wheels.
  - Rotation of wheels result in linear motion of the bicyclist and his bike.

Examples

- Running
  - Coordinate joint rotations to create translation of the entire body.
- Softball pitch
  - Rotate body to achieve desired linear velocity of the ball at release.
- Golf
  - Rotate body to rotate club to strike the ball for intended distance and accuracy.
- Example specific to your interests:

- Key concept:
  - the motion of any point on a rotating body (e.g., a bicycle wheel) can be described in linear terms
- Key information:
  - axis of rotation
  - radius of rotation: distance from axis to point of interest



- <u>Linear and angular displacement</u>

$$d = \theta \, x \, r$$

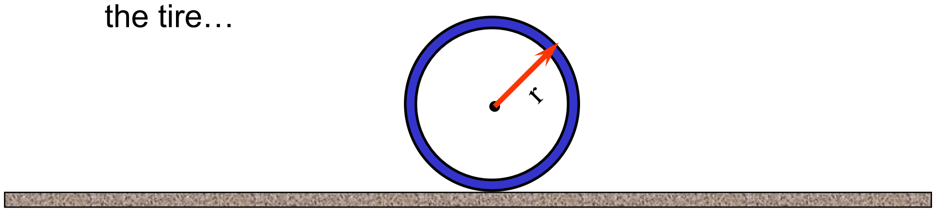
WARNING

θ must be expressed in the units of **<u>radians</u>** for this expression to be valid

<u>NOTE</u>: radians are expressed by a "unit-less" unit. That is, the units of radians seem to be invisible in each of the equations which related linear and angular motion.

Example

- Bicycle odometers measure ***linear distance*** traveled per wheel ***rotation*** for a point on the outer edge of the tire…



- You describe bicycle wheel radius (r = 0.33 m)
- Device counts rotations (θ = 1 rev = 2 π rad)
- Question: How many times did a Tour de France's cyclist's wheel rotate (d = 3427.5 km)?
  - know:
  - need:
  - use:
  - answer:

- <u>Linear and angular velocity</u>

$$v_T = \omega \times r$$

WARNING

ω must be expressed in the units of **<u>radians/s</u>** for this expression to be valid

- Although v_T may appear to be a new term, it is simply the linear or tangential velocity of the point of interest.

Example: Hockey wrist shot

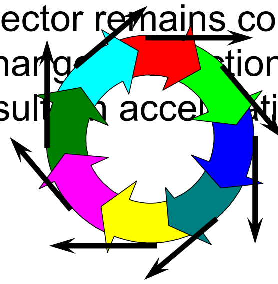
- A hockey player is rotating his stick at 1700 deg/s at the instant of contact. If the blade of the stick is located 1.2 m from the axis of rotation, what is the linear speed of the blade at impact?
  - know:
  - need:
  - use:
  - answer:

Follow up questions

- What would happen to blade velocity if the stick was rotated two times faster?
- What would happen to blade velocity if the stick (radius of rotation) was 25% shorter?

- What does the $v_T = \omega r$ relationship tell us about performance?
  - In many tasks, it is important to maximize the linear velocity (v_T) of a projectile or of a particular endpoint (usually distal)
    - club head speed in golf
    - ball velocity in throwing
  - Theoretically: v_T can be increased in two ways:
    - increasing r
    - increasing ω
  - Problem: it is more difficult to rotate an object when its mass is distributed farther from the axis of rotation.
  - What are some examples of this tradeoff?

- Linear and angular acceleration
  - Newton's 1st law of motion states that an object must be forced to follow a curved path.
  - A change of direction represents a change in velocity (a vector quantity).
  - Therefore, even if the magnitude of a velocity vector remains constant (10 m/s), a change in direction of the velocity vector results in acceleration.



Radial acceleration

- Radial acceleration (a_R) - the linear acceleration that serves to describe the change in direction of an object following a curved path.
  - Radial acceleration is a linear quantity
  - It is always directed inward, toward the center of a curved path.

Example – Radial acceleration

- Skaters or skiers on a curve must force themselves to change directions.
- Changes of direction result in changes in velocity - even if the speed remains constant (why?)
- Changes of velocity, by definition, result in accelerations (a_R).
- This radial acceleration is caused by the component of the ground reaction force (GRF) that is directed toward the center of the turn.

$$a_R = v_T^2/r = (\omega r)^2/r = \omega^2 r$$

- This relationship demonstrates:
  - for a given r, higher v_T is related to a higher a_R; which means a higher force is needed to produce a_R (i.e., to maintain curved path).
  - for a given r, higher w is also related to a higher a_R; which means a higher force is needed to produce a_R (i.e., to maintain curved path).
  - for a given v_T, lower r (i.e., a tighter "turning radius") results in a higher a_R (and the need for a greater force to maintain a curved path)

Example scenarios

Christophe Ena / AP

- Two bicyclists are racing on a rainy day and both enter a slippery corner at 25 m/s. If the one cyclist takes a tighter turning radius than the other, which cyclist experiences the greatest radial acceleration?
  - Who is at greater risk for slipping or skidding?
  - What strategies can cyclists take to reduce the risk of skidding?
  - Which strategy is theoretically more effective?

Other examples

- A baseball pitcher delivers two pitches with exactly the same technique. However, the first pitch is thrown two times faster than the second (e.g. fastball vs very slow change up).
  - During which pitch does the athlete experience greater radial accelerations?
  - In which direction(s) are the radial accelerations experienced?
  - How these accelerations relate to injury (e.g., rotator cuff damage)?
- In preparation for his high-bar dismount, a gymnast increases his rate of rotation by a factor of three. His radius of rotation remains the same.
  - By what factor does his radial acceleration change during this time?

- <u>Tangential acceleration</u> (a_T) - the linear acceleration that serves to describe the rate of change in magnitude of tangential velocity.

$$a_T = (v_{Tf} - v_{Ti})/t$$

- Although a_T may appear to be a new term, it is simply the change in linear or tangential velocity of the point of interest.

Resultant Acceleration Vector

- Rotational and curvilinear motions will always result in radial acceleration because the direction of the velocity vector is always changing.
- If the magnitude of the velocity vector also changes, tangential acceleration will also be present.
- Therefore, during all rotational and curvilinear motions the resultant acceleration is composed of the radial and tangential accelerations.