1. If one homogeneous solution, \( y = u(x) \), of a linear ODE is known, then it may be used to find the remainder of the solutions in the form \( y = u(x)v(x) \), by a method known as reduction of order. Use this method to find the general solution to

\[
(x^2 + 1) \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} + 2y = 0
\]

given that \( y = x \) is a nontrivial solution.

2. Consider the diffusion of solute across a membrane separating two equal chambers:

\[
\frac{dx_1}{dt} = -\frac{1}{\tau} (x_1 - x_2)
\]

\[
\frac{dx_2}{dt} = \frac{1}{\tau} (x_1 - x_2)
\]

where both \( x_1 = x_0 \) and \( x_2 = 0 \) at \( t = 0 \). Solve for the concentrations \( x_1 \) and \( x_2 \) as functions of time.

3. The equation

\[
r^2 \frac{d^2 F}{dr^2} + 2r \frac{dF}{dr} - n(n+1)F = 0
\]

arises when solving LaPlace’s equation in spherical coordinates (later in the course). Find the general solution for \( F(r) \).

4. Find the general solution to

\[
\frac{dy_1}{dx} = 2y_1 - y_2 + y_3 \tag{1}
\]

\[
\frac{dy_2}{dx} = -y_1 + 2y_2 - y_3 \tag{2}
\]

\[
\frac{dy_3}{dx} = y_1 - y_2 + 2y_3 \tag{3}
\]

5. Find the complete solution to

\[
x \frac{dy}{dx} - k \cdot y = x^2,
\]

where \( k \) is an arbitrary constant.

6. Consider the general linear ordinary differential equation \( Ly = f(x) \). Prove that \( y = y_h + y_p \) is the complete solution, where \( y_h \) is the general homogeneous solution of \( L(y_h) = 0 \), and \( y_p \) is any one particular solution of \( L(y_p) = f(x) \). Hint: examine a second particular solution \( y_p' \).