Mass of the Coma Cluster

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Two Different Approaches

• Hughes 1989:
  – Fit to X-ray data from Einstein, EXOSAT, and Tenma
  – Model based on star cluster with and without extra dark matter term

• Kubo, Stebbins, Annis, Dell’Antonio, Lin, Khiabanian, and Frieman 2007
  – Fit to Sloan survey data
  – Fit to NFW galactic cluster model
Hughes’s Approach

• Assume the Coma cluster is spherical, with no subclumps
  – “I have chosen to ignore this complication”

• Assume a virial mass in hydrostatic equilibrium

• Find mass distribution

• Find temperature distribution

• Estimate mass
Fig. 1.—X-ray map of the Coma cluster of galaxies, 0.15–2.0 keV, in 2' × 2' bins. Picture has been smoothed by averaging the observed number of counts in a given bin and that of three adjacent bins. Mirror vignettes by a factor of 2 at 38' off-axis.
Gas Mass Distribution

- Assume isothermal, gas follows light
- Find a best fit to Einstein Observatory data

\[
\rho_g = \rho_{g0} \left\{ 1 - \frac{6}{5} \beta \left[ 1 - (1 + y^2)^{-1/2} \right] \right\}^{2/3}
\]
Hydrostatic Equilibrium

• Uniform density gas in equilibrium:
  \[ \nabla P = -\rho_g \nabla \Phi \n\]

• Assume spherically symmetric and ideal gas:
  \[
  \frac{dT}{dR} = -\frac{1}{\rho_g} \frac{d\rho_g}{dR} T + \frac{4\pi \mu m_H G}{k R^2} \int_0^R drr^2 \rho_b(r)
  \]
Binding Mass Distribution

• Assume either King (1966) model (star cluster) or dark matter power law (n=3,4,5):

\[ \rho_b = \rho_{b0} [(1 + (R/R_b)^2)]^{-n/2} \]

• Which results in:

\[ \frac{dt}{dy} = \frac{3\beta y}{1 + y^2} t - C \frac{1}{y^2} f_n \left( \frac{yR_c}{R_b} \right) \]

\[ f_n(x) = (R_b/R_c)^3 \int_0^x duu^2(1 + u^2)^{-n/2} \]

\[ C = 5.92 \left( \frac{\rho_{b0} h_{50}^2}{10^{-23} \text{ g cm}^{-2}} \right) \left( \frac{kT_0}{10 \text{ keV}} \right)^{-1} \left( \frac{R_c h_{50}^{-1}}{8.5} \right)^2 \]
Temperature Distribution

King Model

$n=4$ polynomial
Mass Follows (X-Ray) Light

\[ \beta = 0.86, R_c = 5.5' \quad \beta = 0.76, R_c = 9.8' \quad \beta = 0.63, R_c = 7.6' \]

\[ P_{bo} \left( 10^{-2.5} \text{ g cm}^{-3} \right) \]

\[ \overline{kT} \text{ (keV)} \]

Graph showing the relationship between \( P_{bo} \) and \( \overline{kT} \).
Mass Follows (X-Ray) Light

<table>
<thead>
<tr>
<th>Band</th>
<th>Model</th>
<th>$10^{-25} \rho_{bo} h_{50}^2 \text{ g cm}^{-3}$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>X-Ray .......</td>
<td>$\beta = 0.86, R_e = 5.5$ (truncated)</td>
<td>$0.98^{+0.021}_{-0.022}$ b</td>
<td>70.7</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0.76, R_e = 9.8$</td>
<td>$0.99^{+0.017}_{-0.020}$ b</td>
<td>75.2</td>
</tr>
<tr>
<td></td>
<td>$\beta = 0.63, R_e = 7.6$</td>
<td>$0.97^{+0.013}_{-0.014}$ b</td>
<td>68.8</td>
</tr>
<tr>
<td></td>
<td><em>Tenma</em> efficiencies $+5%$</td>
<td>$0.93 \pm 0.020^b$</td>
<td>71.3</td>
</tr>
<tr>
<td></td>
<td><em>Tenma</em> efficiencies $-5%$</td>
<td>$1.01 \pm 0.018^b$</td>
<td>69.0</td>
</tr>
<tr>
<td>Optical .......</td>
<td>Merritt</td>
<td>$1.16 \pm 0.12$</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td>The and White</td>
<td>$0.94 \pm 0.14$</td>
<td>...</td>
</tr>
<tr>
<td></td>
<td>Kent and Gunn</td>
<td>$1.14$</td>
<td>...</td>
</tr>
</tbody>
</table>

Not much difference due to calibration of Tenma space telescope.
Binding Mass Distribution

- Dashed line = 99% confidence
- All intersect $1 \times 10^{-25}$
- Not much difference between King and $n=3$
Mass Results

<table>
<thead>
<tr>
<th>Model</th>
<th>$M(R &lt; 1h_{50}^{-1} \text{ Mpc})$ ($10^{14}h_{50}^{-1} M_\odot$)</th>
<th>$M(R &lt; 5h_{50}^{-1} \text{ Mpc})$ ($10^{15}h_{50}^{-1} M_\odot$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ~ Light</td>
<td>5.2–6.7</td>
<td>1.6–2.1</td>
</tr>
<tr>
<td>$n = 3$</td>
<td>5.3–7.1</td>
<td>1.3–3.0</td>
</tr>
<tr>
<td>$n = 4$</td>
<td>5.9–7.0</td>
<td>1.2–2.6</td>
</tr>
<tr>
<td>$n = 5$</td>
<td>6.2–7.2</td>
<td>1.1–2.3</td>
</tr>
<tr>
<td>King: $W_0 = 12$</td>
<td>3.9–5.2</td>
<td>1.6–2.4</td>
</tr>
<tr>
<td>King: $W_0 = 18$</td>
<td>4.1–5.8</td>
<td>1.6–2.2</td>
</tr>
</tbody>
</table>

- $h_{50}^{-1} = 2 \, h^{-1}$
- Range: $0.6-1.5 \times 10^{15} \, h^{-1}$
Kubo et al.’s Approach

• Select a set of galaxies from SDSS
• Measure tangential shear across those galaxies
• Convert tangential shear to a surface density
• Fit that density to a Navarro, Frenk, and White (NFW) profile
• Convert the density profile to a virial mass
Observations

• Galactic data from the Sloan Digital Sky Survey Data Release 5 (SDSS DR5)
• Used PHOTO pipeline to do object detection and shape measurement
• 200 deg$^2$ region of the sky centered at NGC 4889 (13$^h$02$^m$0.2$^s$, 27°41′26.6″, J2000)
Sample Selection

- Only objects PHOTO identified as galaxies
- Also cut ones with bad PSF
- \( \sim 270,000 \) galaxies total
Tangential Shear

• Break into radial bins from 0.05-10.5 $h^{-1}$ Mpc

• Use 1-D shear based on Castro et al. (2005):

$$\gamma_t = \frac{1}{2\mathcal{R}} \frac{\sum e_t}{N}$$

• $\gamma_t =$ tangential shear for bin, $e_t =$ tangential ellipticity of bin, $N =$ number in bin

$$\mathcal{R} = 1 - \sigma^2_{SN}$$
Tangential Shear

- Squares = $\gamma_t$ with 1 $\sigma$ error
- Triangles = 45° shear
- Total $\chi^2 = 23.33$
Surface Density

• Tangential shear relates to surface density:

\[ \gamma_t = \frac{\bar{\Sigma}(\leq r) - \Sigma(r)}{\Sigma_{\text{crit}}} \]

\[ \Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}} \]

• \( D_s, D_l \) angular diameter distance to source or lens; \( D_{ls} \) angular distance from lens to source
NFW Profile

• Fit to profile from Navarro, Frenk, and White:

\[ \rho(r) = \frac{\delta_c \rho_c}{(r/r_s)(1 + r/r_s)^2} \]

\[ \delta_c = \frac{200}{3} \frac{c^3}{\ln (1 + c) - c/(1 + c)} \]

\[ \rho_c = \frac{3H^2(z)}{8\pi G} \]

• The two parameters, \( r_s \) and \( c \) can be constrained by setting the virial radius \( r_{200} = c r_s \):

\[ M_{200} = \frac{800\pi}{3} \rho_c r_{200}^3 \]
Results!

• Fitting the profile:

\[ r_{200} = 1.99^{+0.21}_{-0.22} \, h^{-1} \, \text{Mpc} \]
\[ c = 3.84^{+13.16}_{-1.84} \]

• With \( \chi^2 = 3.87 \) for 4 degrees of freedom

• For a mass of:

\[ M_{200} = 1.88^{+0.65}_{-0.56} \times 10^{15} \, h^{-1} \, M_\odot \]
## Cross-Comparison

<table>
<thead>
<tr>
<th>Virial Radius $(h^{-1} \text{ Mpc})$</th>
<th>Mass $(10^{15} h^{-1} M_\odot)$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.99^{+0.21}_{-0.22}$</td>
<td>$1.88^{+0.65}_{-0.56}$</td>
<td>1&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>1.5</td>
<td>0.8</td>
<td>2&lt;sup&gt;a&lt;/sup&gt;</td>
</tr>
<tr>
<td>2.5</td>
<td>$0.93 \pm 0.12$</td>
<td>3</td>
</tr>
<tr>
<td>2.7</td>
<td>$0.95 \pm 0.15$</td>
<td>4</td>
</tr>
</tbody>
</table>

1. Kubo et al. 2007  
2. Geller et al. 1999

1. Hughes 1989  
2. The & White 1986
Questions?