Pair-Instability Black Hole Formation and You!

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Hyper-Large Pop III Stars

- Population III may have been home to a number of ridiculously large stars (100-10,000 M☉)
- Such stars would have practically zero metalliclicity, and thus loose little mass during main sequence
- Internal pressures high enough to create pair-instability
Pair Instability

Main Sequence Fusion

Increased Gamma Emission

Pair Production

Increased Fusion

Feedback Loop

Neutrinos

Feedback Loop

Neutrinos
Pair Instability

Initial Stellar Mass > 100 M☉

\[ e^- / e^+ \text{ Pair Instability} \]

\(\text{ISM} < 260 \text{ M}_☉\)

Pair Instability Supernova

\(\text{ISM} > 260 \text{ M}_☉\)

Neutrino Burst and Black Hole Formation
Science Objective

• Goal of Nakazato et al. was to model the spherically symmetric gravitation collapse of Pop III massive stars

• From this, a relic neutrino background flux can estimated, providing a direct measurement of Population III
Pop III Stars Modeled

- 18 ISMs modeled
- ISM > 260 $M_\odot$ (Black hole formation)
- ISM < 1600 $M_\odot$ (GR < Pair Instability)

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The Numerical Model

- Model uses spacetime metric from Misner & Sharp (1964):

\[ ds^2 = e^{2\phi(t,m)} c^2 dt^2 - e^{2\lambda(t,m)} \left( \frac{G}{c^2} \right)^2 dm^2 - r^2(t,m)(d\theta^2 + \sin^2 \theta d\phi^2), \]

- Energy equation:

\[ e^{-\phi} \left( \frac{\partial \varepsilon}{\partial t} \right)_m = - \frac{p}{\Gamma} \frac{\partial}{\partial m} (4\pi r^2 U), \]

\[ U = e^{-\phi} \frac{\partial r}{\partial t}, \]

\[ \Gamma = e^{-\lambda} \frac{\partial r}{\partial m}, \]
Neutrino Reactions

\[ \nu_e + n \leftrightarrow e^- + p, \]
\[ \bar{\nu}_e + p \leftrightarrow e^+ + n, \]
\[ \nu + N \leftrightarrow \nu + N, \]
\[ \nu + e^- \leftrightarrow \nu + e^-, \]
\[ N + N' \leftrightarrow N + N' + \nu + \bar{\nu}, \]
\[ \gamma^* \leftrightarrow \nu + \bar{\nu}, \]
\[ e^- + e^+ \leftrightarrow \nu + \bar{\nu}, \]
\[ \nu + A \leftrightarrow \nu + A, \]
\[ \nu_e + A \leftrightarrow A + e^-, \]
Core Collapse

• Start with a reference hydrodynamic/GR model for density and temperature
Core Collapse

- Core is divided into two parts:
  - Subsonic inner core ($U \sim r$)
  - Supersonic outer core ($U \sim r^{-1/2}$)
Apparent Horizon

• To chart black hole formation, the model tracks the trapped surfaces described by:

\[ U + \Gamma < 0,\]

• Which is satisfied by:

\[ r < r_g \equiv 2\tilde{m},\]

\[ \Gamma^2 = 1 + U^2 - \frac{2\tilde{m}}{r}. \]
Importance of Neutrino Cooling

- Neutrino cooling has a massive effect on core collapse
Importance of Neutrino Cooling

- With neutrino cooling, core shock disappears and entropy drops.

![Graph showing the relationship between baryon mass and entropy before and after neutrino cooling.](image-url)
Electron Fraction

• The high entropy in the core prevents the core from reaching electron degeneracy pressure.
• The positron capture rate is slower than electrons.
• Equilibrium is reached where $\beta$ / inverse $\beta$ reaction rates equal out
Electron Fraction

![Graphs showing electron fraction ($Y_e$) and proton fraction ($Y_p$) as a function of baryon mass ($M_\odot$).]
Neutrino Luminosity

- Because of the high reaction rates, the neutrino luminosity peaks near $\sim 10^{54}$ erg/s, 10 times higher than a normal supernova.
- But the apparent horizon closes within 100 ms, so total energy emitted is only $\sim 10^{53}$ erg, comparable to a supernova.
Neutrino Luminosity

e\textsuperscript{-} neutrino

\tau, \mu neutrino

e\textsuperscript{+} neutrino

Total neutrino
Neutrino Luminosity

e^− neutrino, 
\( t = -12.3 \text{ ms} \)

\( \tau, \mu \) neutrino, 
\( t = -12.3 \text{ ms} \)

e^+ neutrino, 
\( t = -12.3 \text{ ms} \)

\( \tau, \mu \) neutrino, 
\( t = -1.52 \text{ ms} \)
Neutrino Spectrum

• Because of the high reaction rates, the neutrino luminosity peaks near $\sim 10^{54}$ erg/s, 10 times higher than a normal supernova
• But the apparent horizon closes within 100 ms, so total energy emitted is only $\sim 10^{53}$ erg, comparable to a supernova
Neutrino Energy Spectrum

$e^-$ neutrino

$e^+$ neutrino

$\tau$, $\mu$ neutrino

Total neutrino
Initial Mass Dependence
(or lack thereof)

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Initial Mass Dependence (or lack thereof)

375 M⊙

10,500 M⊙
Initial Mass Dependence (or lack thereof)

- The spectrum does not change drastically over the mass range
Relic Neutrino Flux

• The net flux is an integral over the number density of Pop III stars and their redshift:

\[
\frac{dF_\nu}{dE_\nu} = c \int_{z_i}^{z_f} \int_{M_0}^{M_N} \frac{dN(m, E'_\nu)}{dE'_\nu} (1 + z) R_{\text{PopIII}}(z, m) dm \frac{dt}{dz} \, dz,
\]

• Substituting for redshift and adding a normalizing factor \(\Psi(z)\):

\[
\frac{dF_\nu}{dE_\nu} = c \int_{M_0}^{M_N} dm \frac{dn(m)}{dm} \times \int_{z_f}^{z_i} dz \frac{\psi(z)}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \frac{dN(m, E'_\nu)}{dE'_\nu} (1 + z).
\]
Relic Neutrino Flux

• Using a Pop III mass distribution from Nakamura & Umemura (2001)

\[
\begin{aligned}
    \frac{dn}{dm} &= B m^{-\beta - 1}, & m &\geq M_{\text{min}}, \\
    n &= 0, & m &< M_{\text{min}}.
\end{aligned}
\]

\[B = (\beta - 1) M_{\text{min}}^{\beta - 1} n_b m_N \epsilon (1 - \kappa).\]

• So,

\[
\int_{M_{k-1}}^{M_k} \frac{dn(m)}{dm} \, dm = \frac{\beta - 1}{\beta} n_b m_N \epsilon (1 - \kappa) M_{\text{min}}^{\beta - 1} \left( M_{k-1}^{-\beta} - M_k^{-\beta} \right),
\]
Relic Neutrino Flux

• Putting it all together:

\[
\frac{dF_\nu}{dE_\nu} = \frac{\beta - 1}{\beta} c n_b m_N \epsilon (1 - \kappa) M_{\text{min}}^{\beta - 1} \sum_{k=1}^{N} \left( M_{k-1}^{-\beta} - M_k^{-\beta} \right)
\times \int_{z_f}^{z_i} \frac{\psi(z)}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \frac{dN(M_k, E'_\nu)}{dE'_\nu} (1+z),
\]

• But we still need a $\Psi(z)$!
Relic Neutrino Flux

• Model A assumes reionization at $z = 17 \pm 5$ based on WMAP data (Spergel 2003)

\[
\psi(z) = \psi_A(z) \\
\equiv \frac{1}{5\sqrt{2\pi}} \exp\left[-\frac{(z - 17)^2}{20}\right] H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}.
\]

• Model B assumes reionization at $z \sim 10$ (Scannapieco et al., 2003)

\[
\psi(z) = \psi_B(z) \equiv \delta(z - 10)H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}.
\]

• Model C just assumes continuous Pop III formation across $z = 4-12$ (Yonetoku, 2004)

\[
\psi(z) = \psi_C(z) \propto (1+z)^{1.7} \quad \text{for} \quad 4 < z < 12,
\]
Results!

- Assuming Model A (WMAP):
Results!

- Anti-electron neutrino flux is relatively high!
Detection?

- Solar neutrinos will overwhelm electron neutrinos below 18 MeV, and ordinary supernovas above 10 MeV
- Terrestrial nuclear reactors will dominate anti-electron neutrinos below 10 MeV
- So, no possible detection, *yet*...
Summary

• Pop III stars greater than 260 $M_\odot$ would form black holes due to pair instability.
• Such a process would produce an intense neutrino burst, with an energy distribution independent of initial mass.
• These neutrinos would create a relic background source.
• But which is below current limits of detection.