Cosmological Applications of Gravitational Lensing

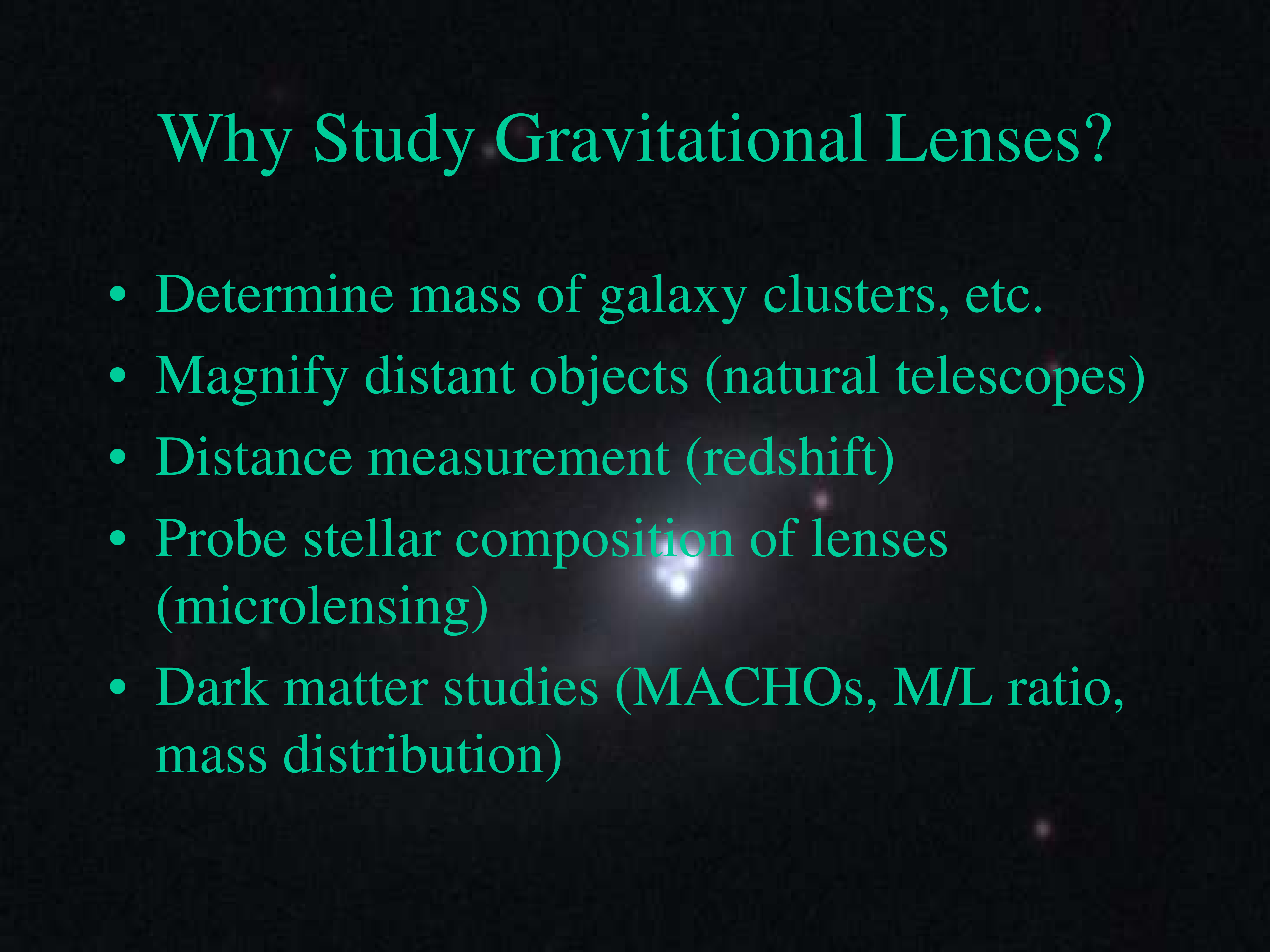
R. D. Blandford and R. Narayan

Annu. Rev. Astron. Astrophys. 1992. 30: 311-58

Presented by Jim Haldenwang

AST 494 / 591 Spring 2007 Dr. Jansen

Why Study Gravitational Lenses?



- Determine mass of galaxy clusters, etc.
- Magnify distant objects (natural telescopes)
- Distance measurement (redshift)
- Probe stellar composition of lenses (microlensing)
- Dark matter studies (MACHOs, M/L ratio, mass distribution)

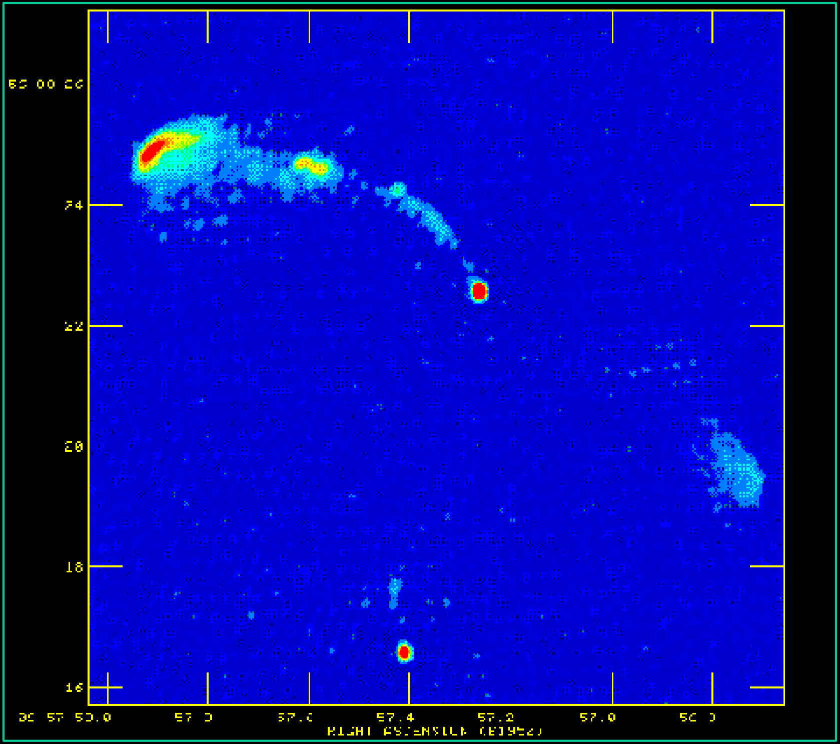
Observational Difficulties

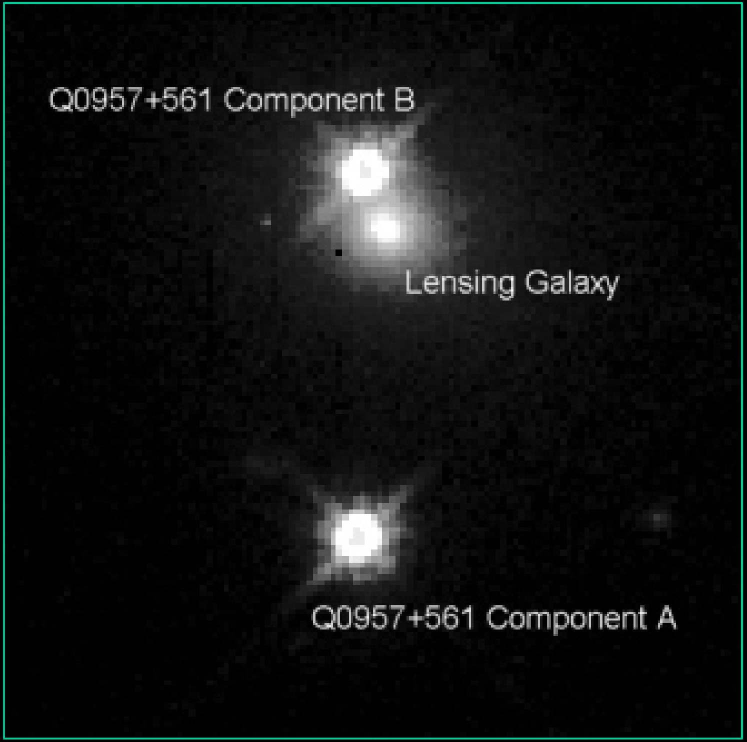
- Rare
- Nature of source disguised by large magnification
- Uncertain lens distribution
- Different source regions magnified to different degrees
- Distortions due to perturbations along line of sight

Classes of gravitational lenses

- Multiple Quasars
  - double, triple, quadruple images
- Arcs
  - source: high redshift galaxy
  - lens: galaxy cluster
- Radio Rings
  - source: extended radio source
  - lens: galaxy

Q0957+561 - Double Quasar

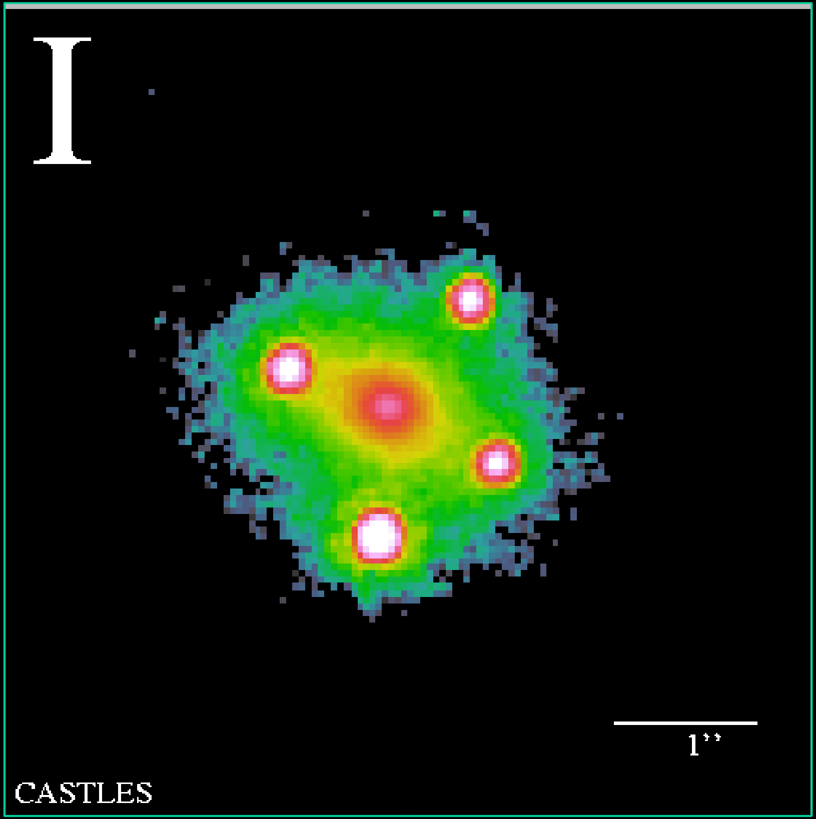




First lens discovered - Walsh et al 1979

source: $z = 1.41$ quasar; lens: $z = 0.36$ galaxy

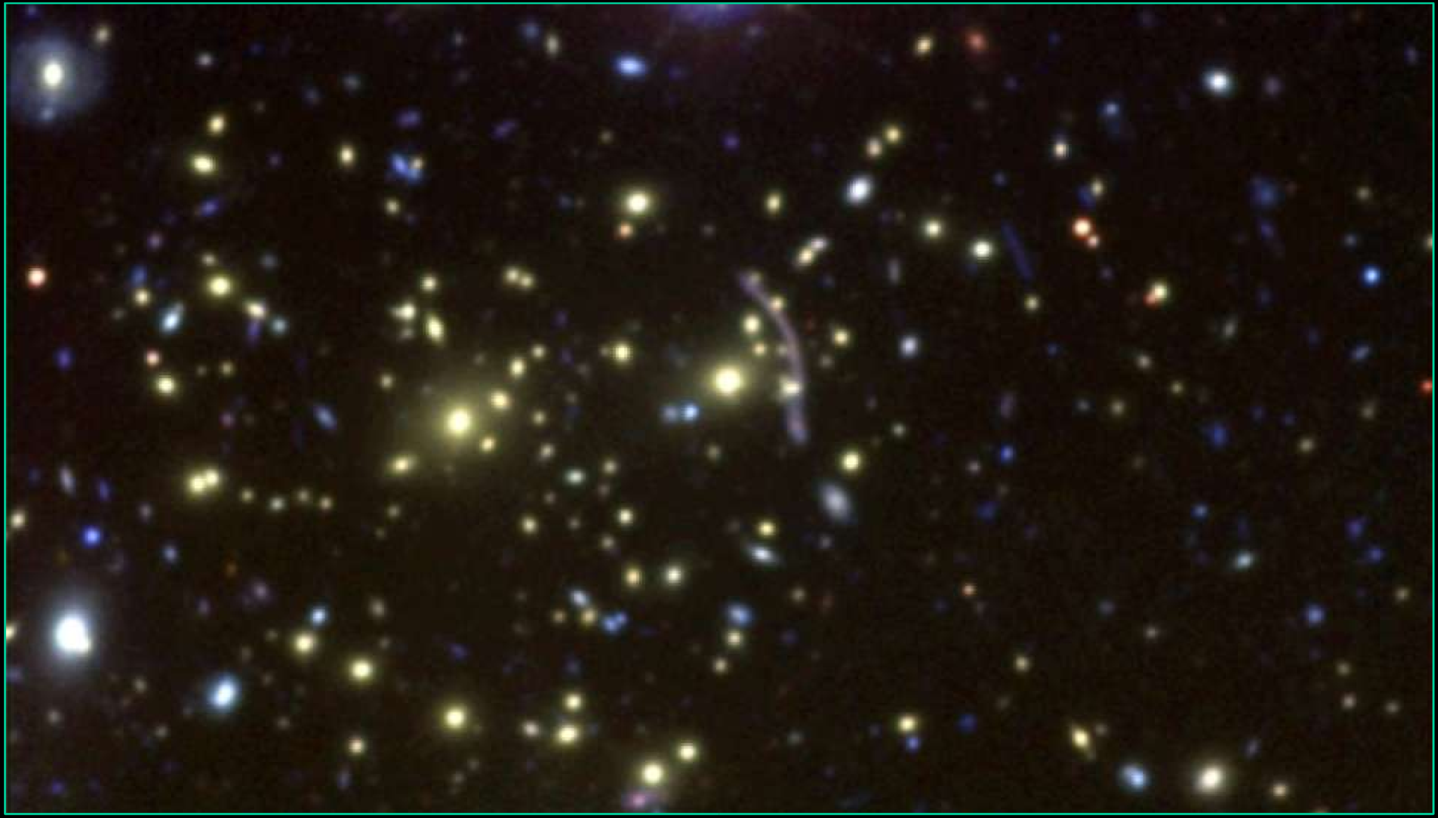
Q2237+030 - Einstein Cross



Quadruple quasar

source: z = 1.69 quasar; lens: z = 0.039 galaxy

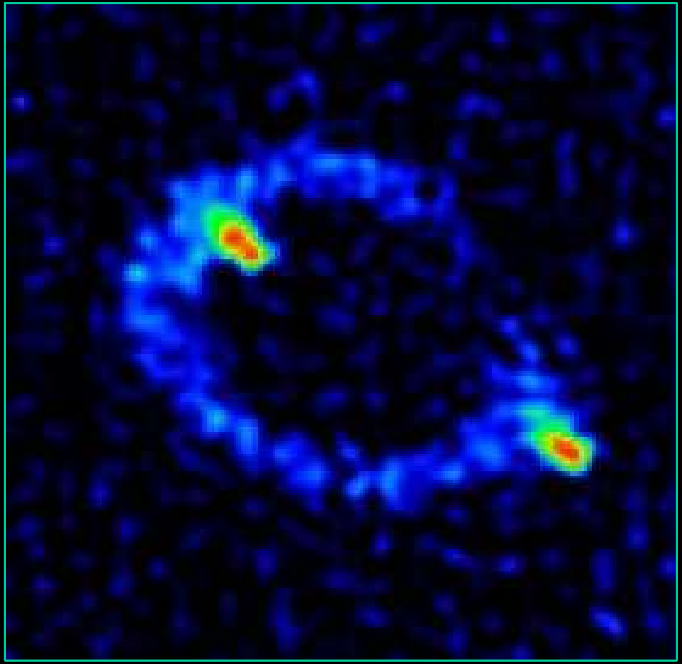
Abell 370 - Arc



Lynds & Petrosian, 1986; Soucail et al, 1987

source: z = 0.72 galaxy; lens: z = 0.37 cluster

MG1131+0456 - Radio Ring

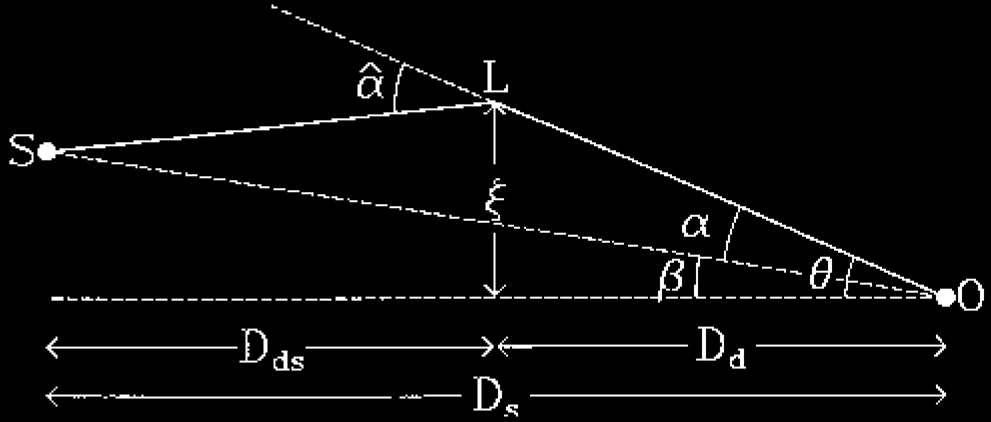


“Einstein ring” discovered by Hewitt et al, 1988

source: z = ? compact radio source

lens: z = ? galaxy

Gravitational Lens Optics



S, L, O - Source, Lens, Observer

D_{ds}, D_d, D_s - angular diameter distances

α - reduced deflection angle

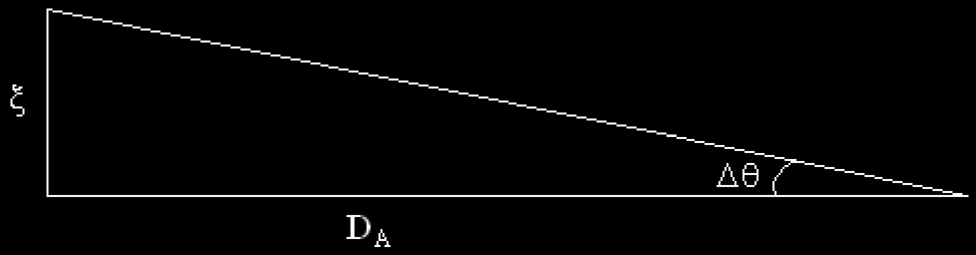
β - source position

θ - image position

ξ - impact (or collision) parameter

$\hat{\alpha}$ - deflection angle (a two-vector)

Angular Diameter Distances

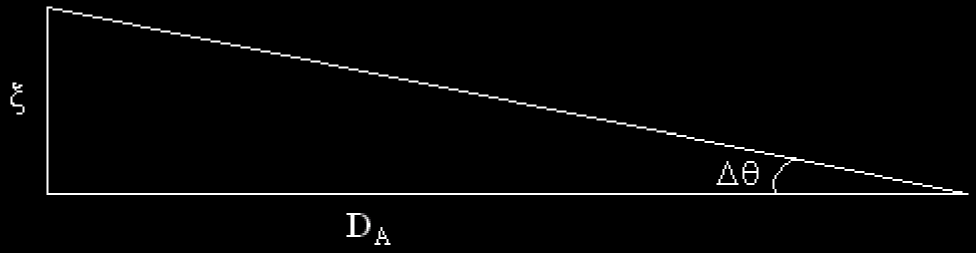


In Euclidean geometry (for small angles),

$$\xi = D_A \cdot \Delta\theta$$

But what about the non-Euclidean geometry of the Friedmann-Robertson-Walker universe?

Angular Diameter Distances



Consider the Robertson-Walker metric:

$$ds^2 = -c^2 dt^2 + a(t)^2 \left[dr^2 + S_\kappa(r)^2 d\Omega^2 \right]$$

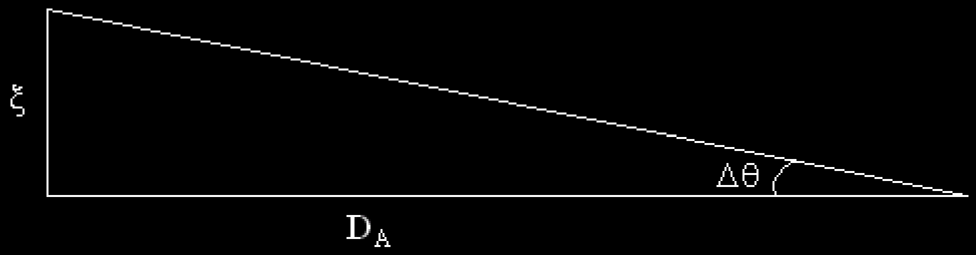
where

$$d\Omega^2 \equiv d\theta^2 + \sin^2\theta \; d\phi^2$$

and

$$S_\kappa(r) = \begin{cases} R\sin(r/R) & (\kappa = +1) \\ r & (\kappa = 0) \\ R\sinh(r/R) & (\kappa = -1) \end{cases}$$

Angular Diameter Distances

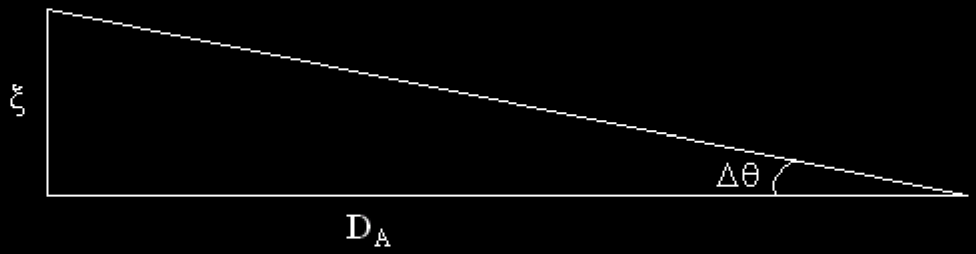


Setting dt = dr = dϕ = 0 in the Robertson-Walker metric, we obtain

$$\xi = \Delta s = a(t_e) \cdot S_\kappa(r) \cdot \Delta\theta$$

For example, in a Euclidean universe, $a(t_e) = 1$ and $S_\kappa(r) = r$, so $\xi = r \cdot \Delta\theta$.

Angular Diameter Distances



In the FRW universe,

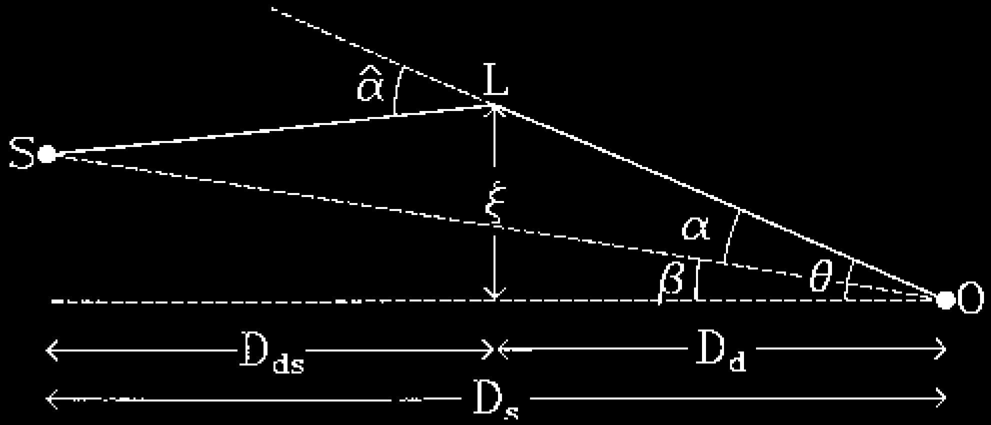
$$\xi = a(t_e) \cdot S_\kappa(r) \cdot \Delta\theta$$

Now we <u>define</u> the angular diameter distance D_A as

$$D_A = a(t_e) \cdot S_\kappa(r) = \frac{S_\kappa(r)}{1+z}$$

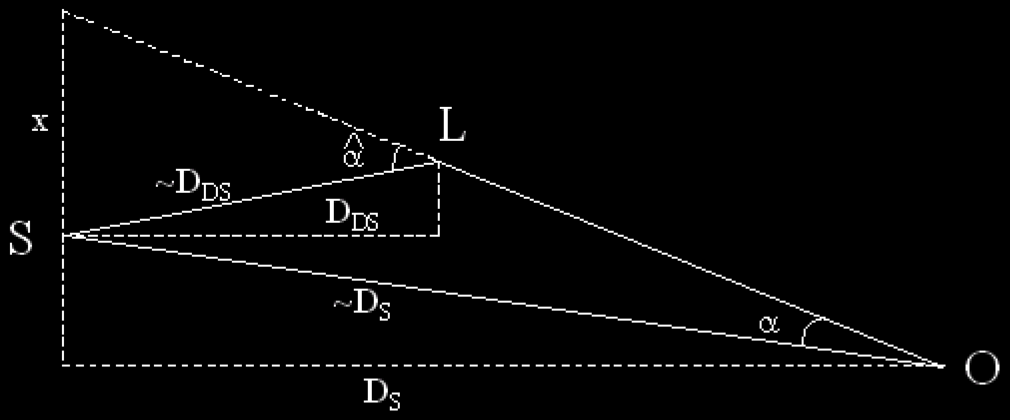
Then $\xi = D_A \cdot \Delta\theta$, just like in the Euclidean universe.

Gravitational Lens Optics



$$\boxed{\alpha = \frac{D_{ds}\hat{\alpha}}{D_s}}$$

Gravitational Lens Optics



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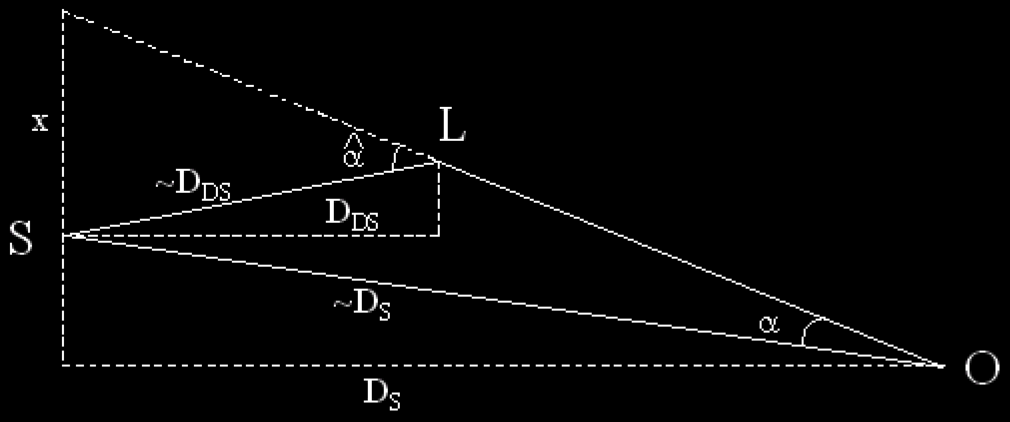

$$\hat{\alpha} \approx \frac{x}{D_{DS}} \qquad \alpha \approx \frac{x}{D_S}$$

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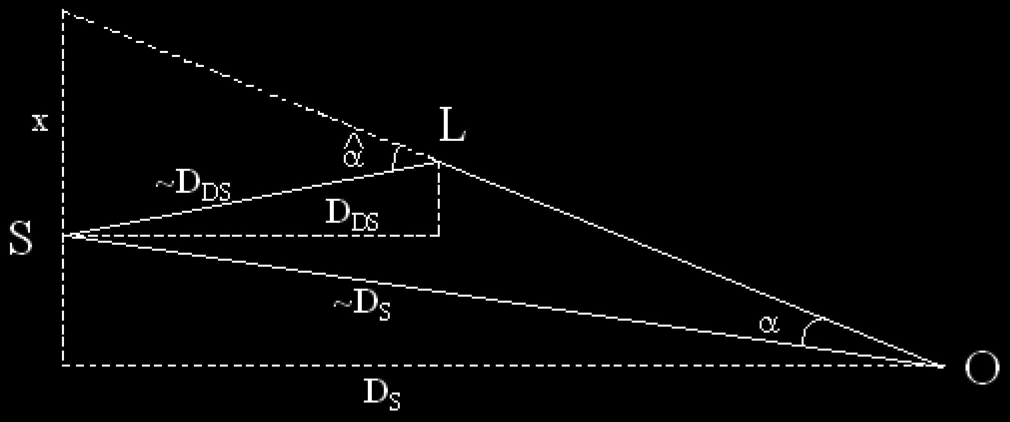
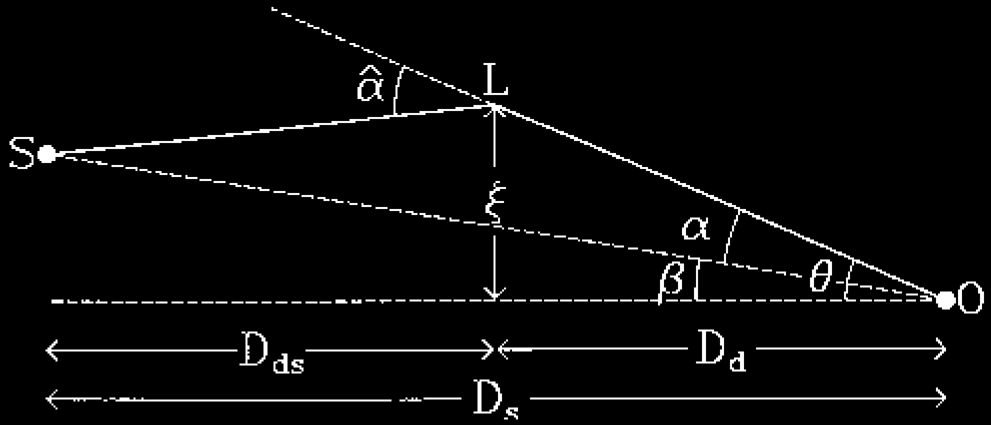


$$\hat{\alpha} \approx \frac{x}{D_{DS}} \qquad \alpha \approx \frac{x}{D_S}$$

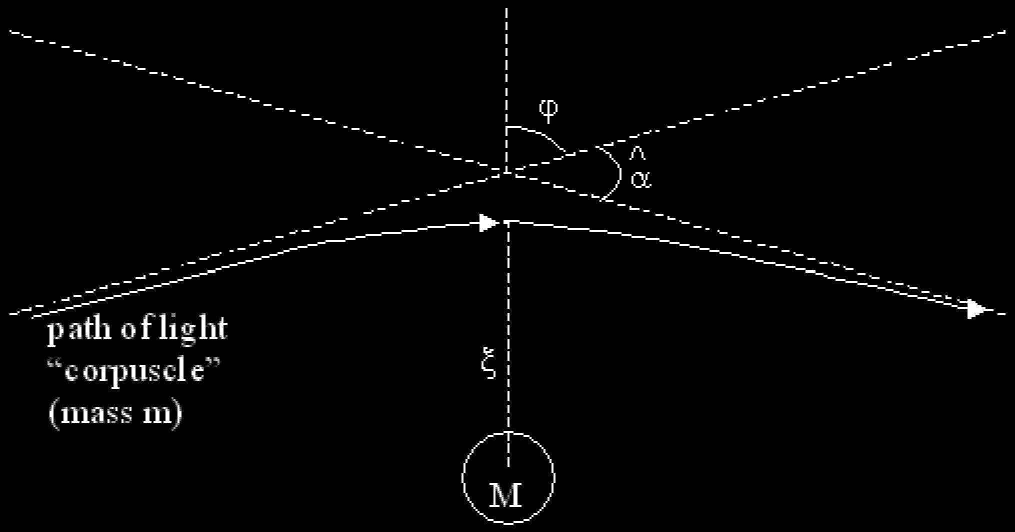
$$x \approx \hat{\alpha} D_{DS} \approx \alpha D_S, \quad \text{so} \quad \alpha \approx \frac{D_{DS}\hat{\alpha}}{D_S}$$

Gravitational Lens Optics

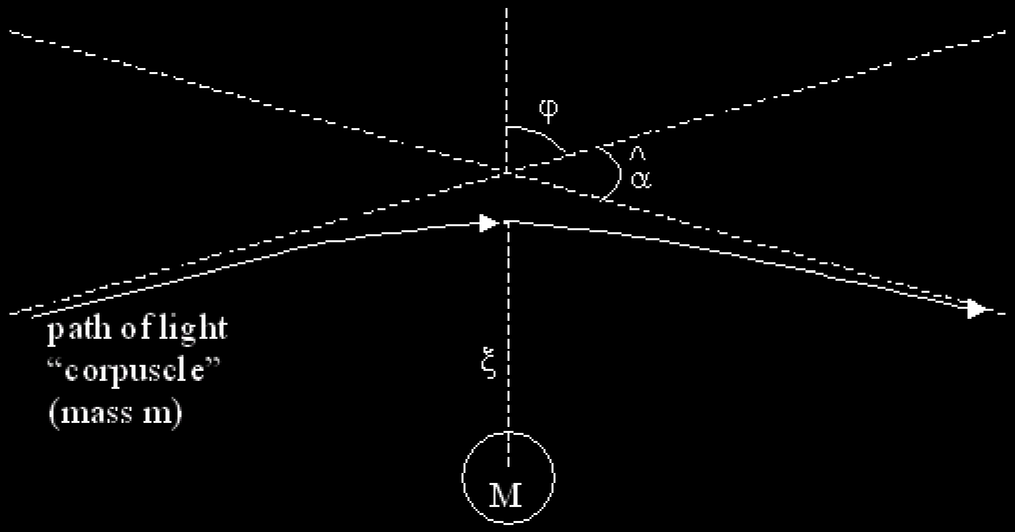


$$\boxed{\hat{\alpha} = \frac{4GM}{c^2\xi}}$$

Newtonian Deflection of Light



Newtonian Deflection of Light

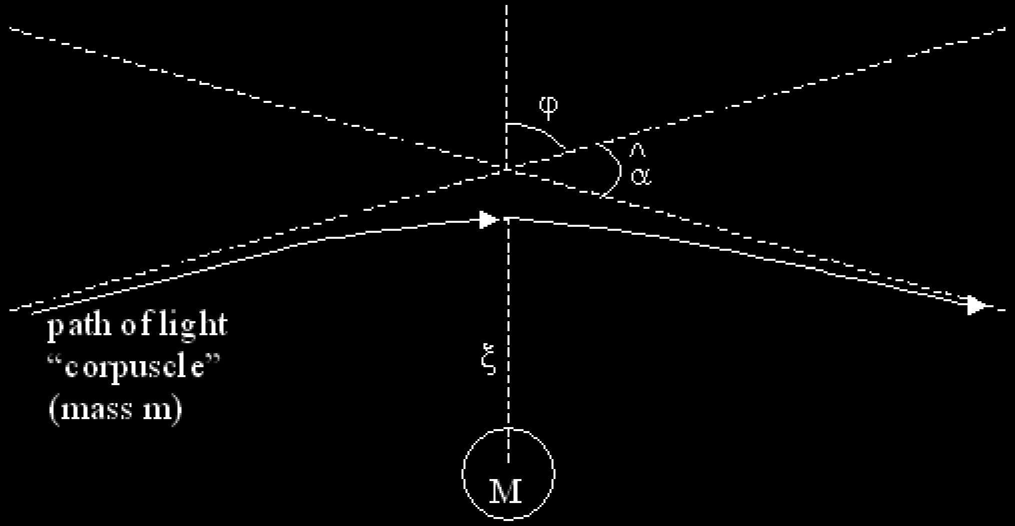


From Analytical Mechanics, 7th edition, by Fowles & Cassiday, eq (6.10.8):

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{G^2M^2m^3}} \qquad \text{where} \qquad E = \frac{1}{2}mv^2 - \frac{GMm}{\xi}$$

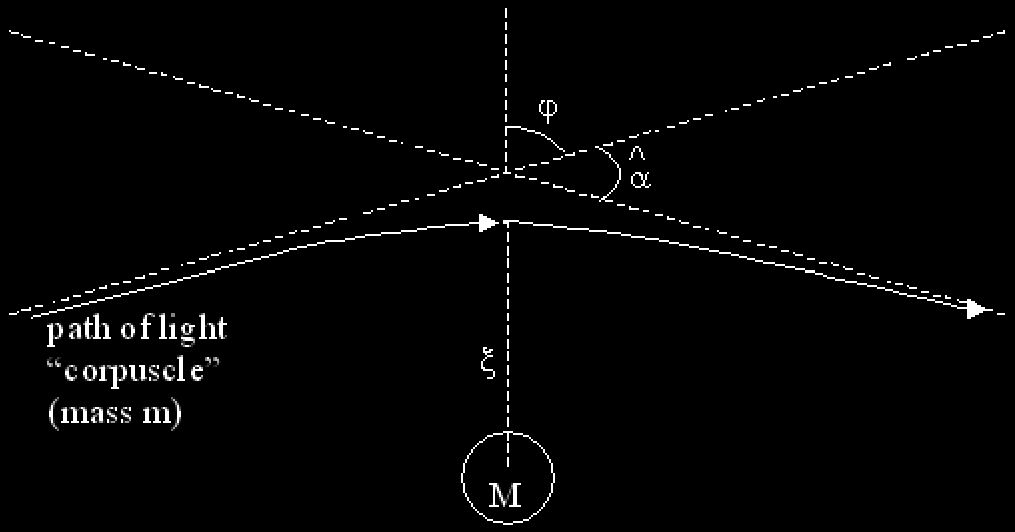
$$\text{and} \qquad L = \xi mv$$

Newtonian Deflection of Light



$$\varepsilon=\sqrt{1+\frac{\xi^2 v^4}{G^2M^2}-\frac{2\xi v^2}{GM}}=\sqrt{\left(1-\frac{\xi v^2}{GM}\right)^2}=\frac{\xi v^2}{GM}-1$$

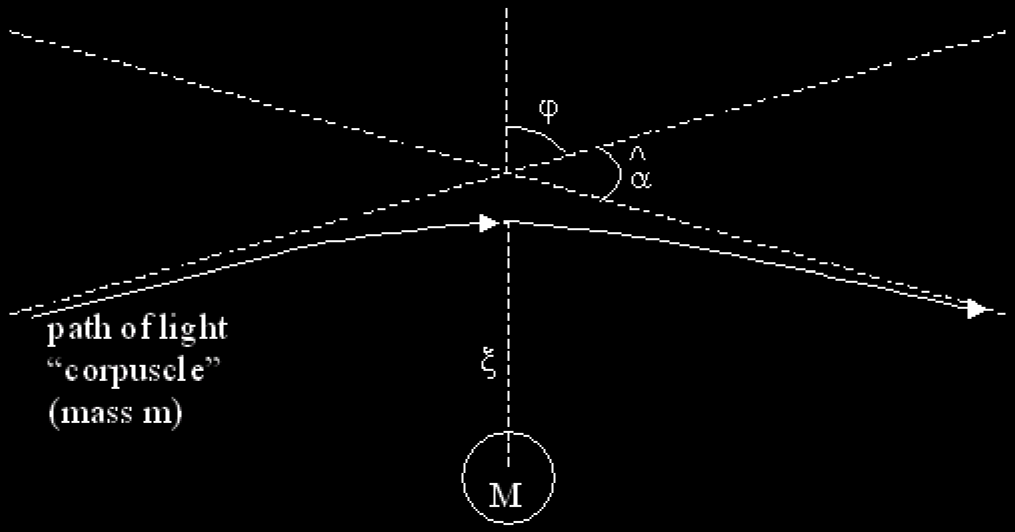
Newtonian Deflection of Light



$$\varepsilon = \sqrt{1 + \frac{\xi^2 v^4}{G^2 M^2} - \frac{2\xi v^2}{GM}} = \sqrt{\left(1 - \frac{\xi v^2}{GM}\right)^2} = \frac{\xi v^2}{GM} - 1$$

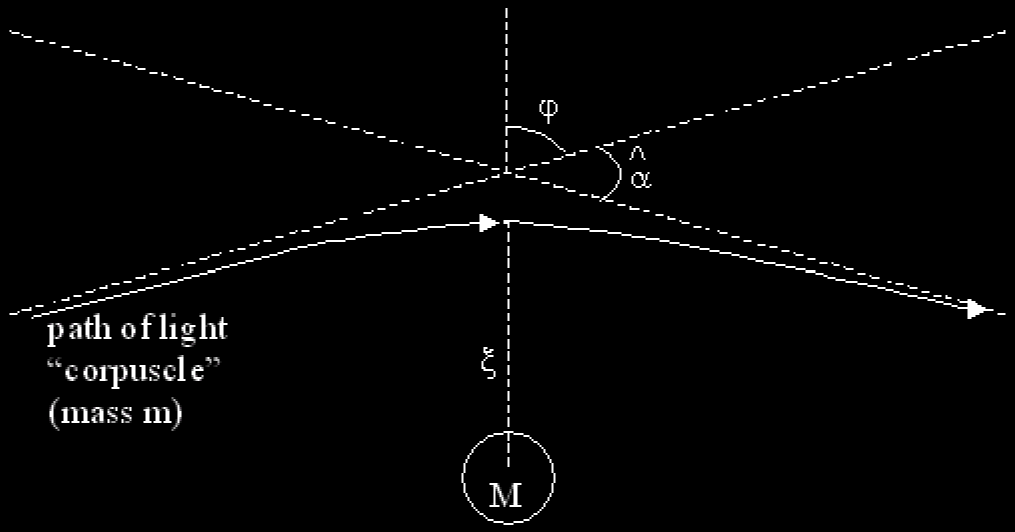
$\varepsilon \approx \frac{c^2 \xi}{GM}$ since $\frac{c^2 \xi}{GM} >> 1$

Newtonian Deflection of Light



$$\cos(\varphi) = \frac{1}{\varepsilon} \Rightarrow \varphi = \cos^{-1}\left(\frac{GM}{c^2\xi}\right)$$

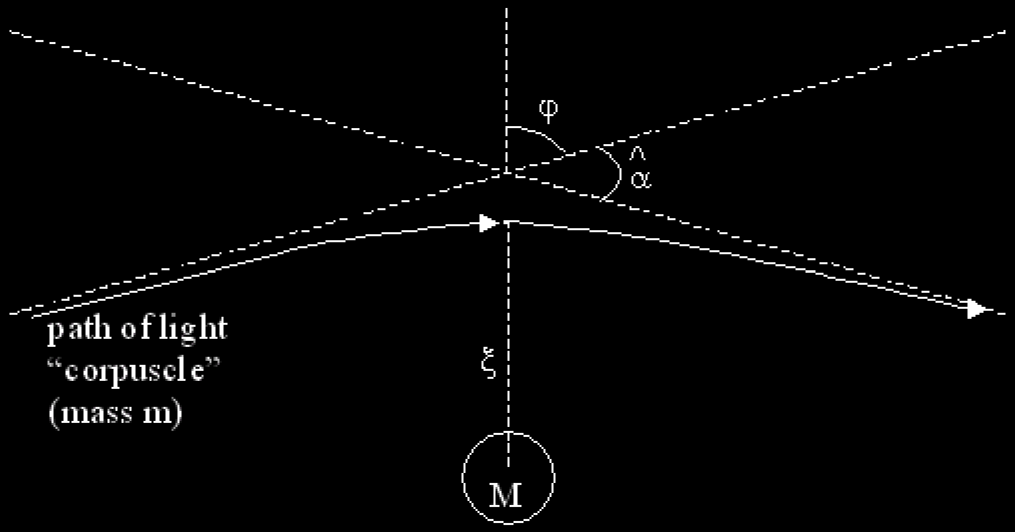
Newtonian Deflection of Light



$$\cos(\varphi) = \frac{1}{\varepsilon} \quad \Rightarrow \quad \varphi = \cos^{-1}\left(\frac{GM}{c^2\xi}\right)$$

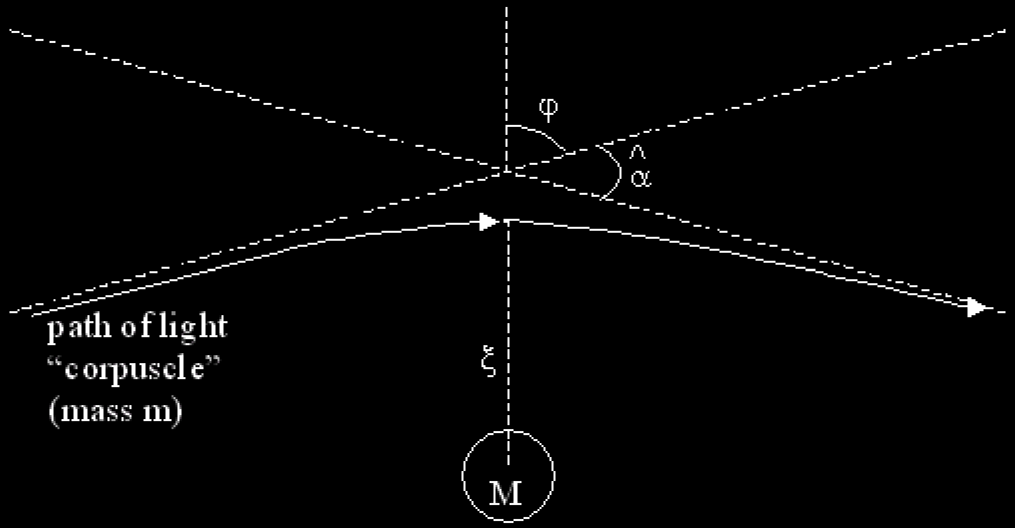
$$\hat{\alpha} = \pi - 2\varphi = 2\left(\frac{\pi}{2} - \varphi\right) \approx 2\left[\frac{\pi}{2} - \left(\frac{\pi}{2} - \frac{GM}{c^2\xi}\right)\right]$$

Newtonian Deflection of Light



$$\hat{\alpha} = \frac{2GM}{c^2 \xi}$$

Newtonian Deflection of Light



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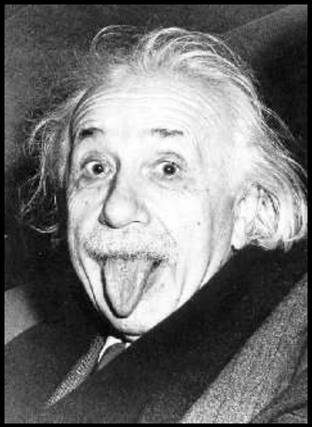

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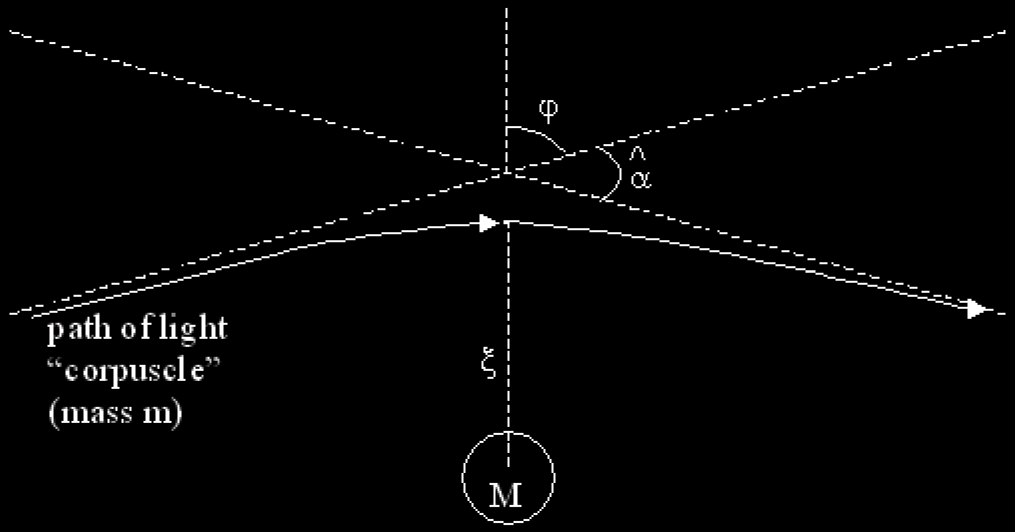

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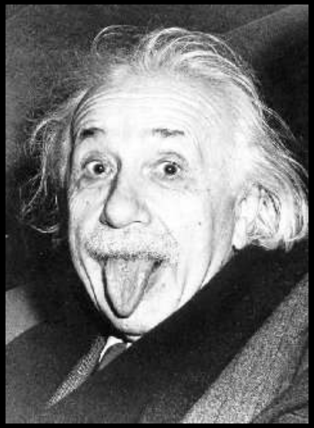

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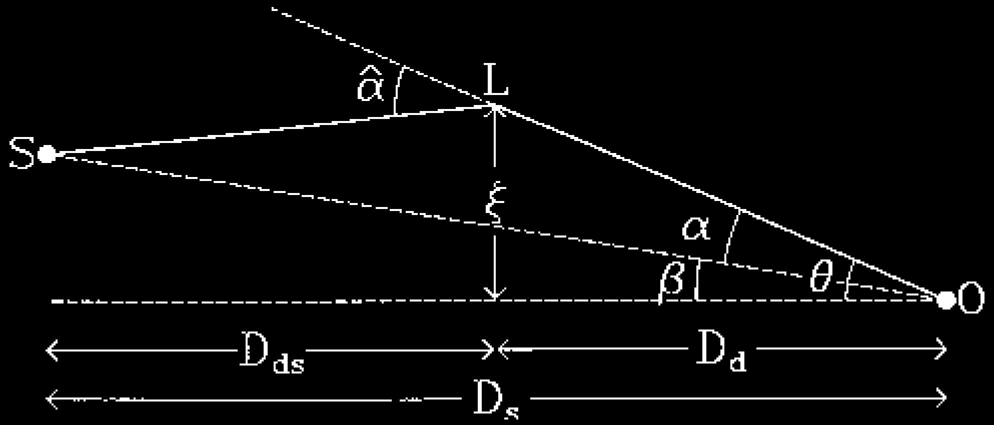
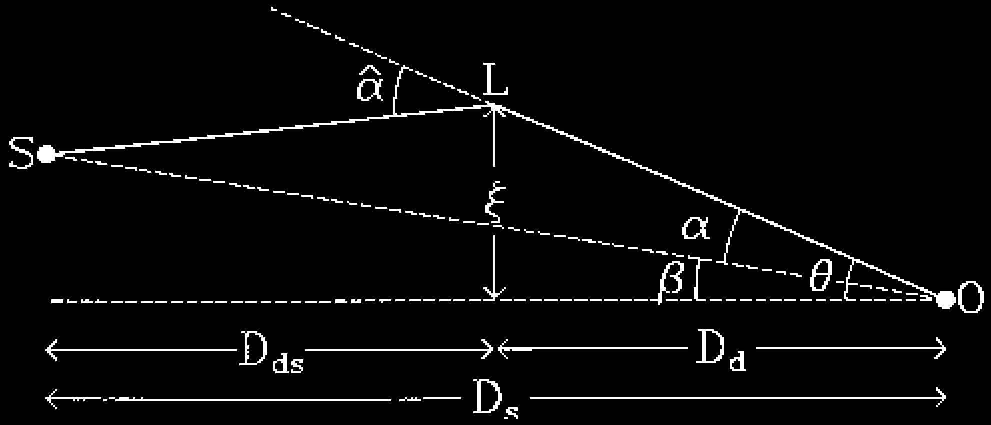
Gravitational Lens Optics



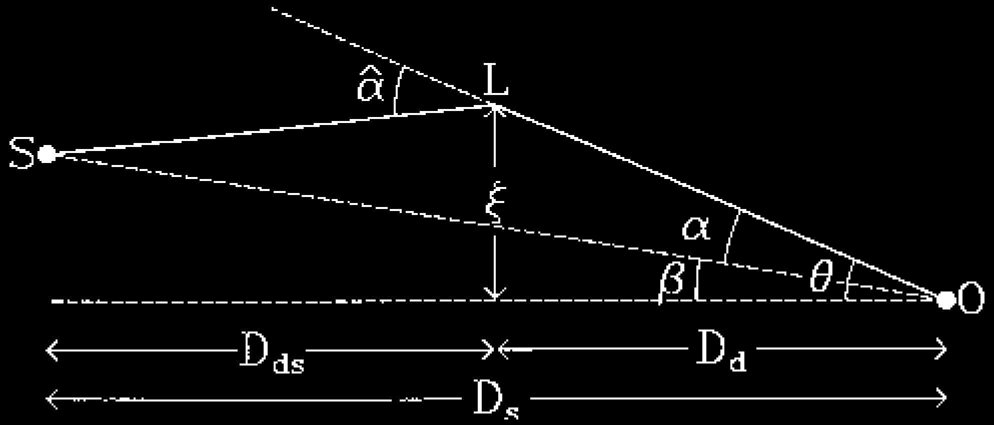
$$\boxed{\theta_E = \left(\frac{4GM}{c^2 D}\right)^{1/2}}$$

Gravitational Lens Optics



$$\alpha = \frac{D_{ds}\hat{\alpha}}{D_s} \Rightarrow \hat{\alpha} = \alpha \frac{D_s}{D_{ds}}$$

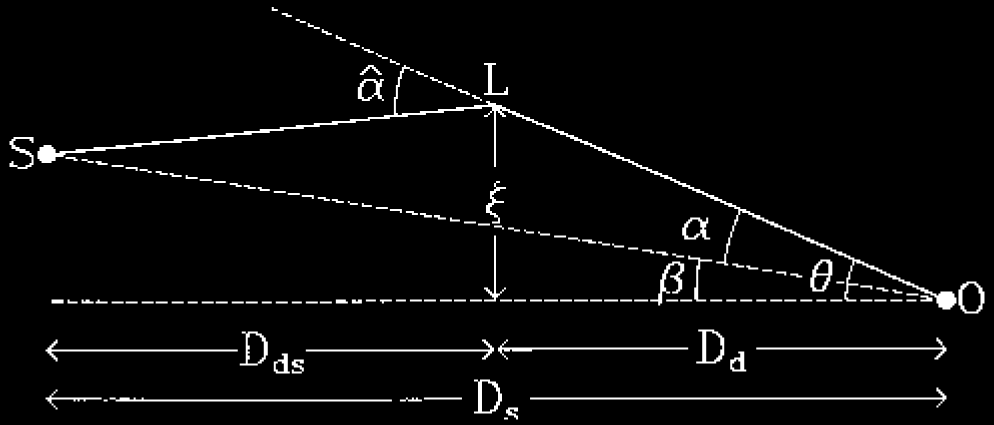
Gravitational Lens Optics



$$\alpha = \frac{D_{ds}\hat{\alpha}}{D_s} \Rightarrow \hat{\alpha} = \alpha \frac{D_s}{D_{ds}}$$

$$\hat{\alpha} = \frac{4GM}{c^2 \xi} \Rightarrow \alpha \frac{D_s}{D_{ds}} = \frac{4GM}{c^2 \xi}$$

Gravitational Lens Optics

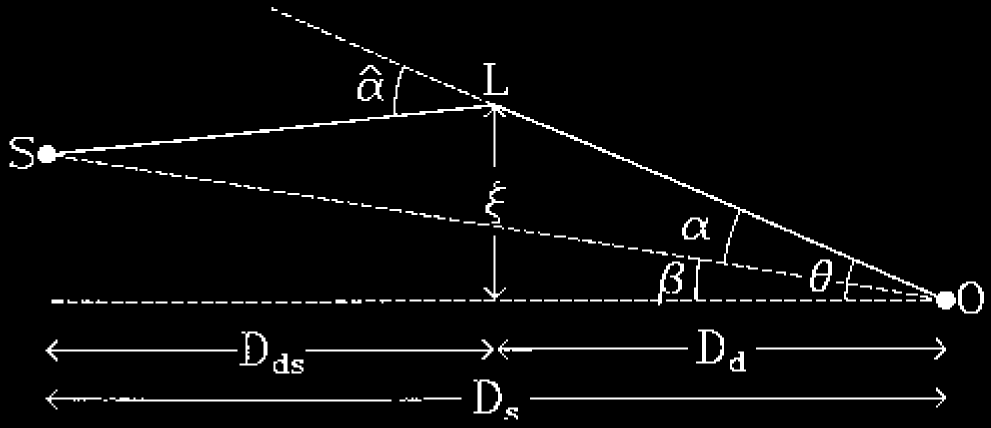


$$\alpha = \frac{D_{ds}\hat{\alpha}}{D_s} \Rightarrow \hat{\alpha} = \alpha \frac{D_s}{D_{ds}}$$

$$\hat{\alpha} = \frac{4GM}{c^2\xi} \Rightarrow \alpha \frac{D_s}{D_{ds}} = \frac{4GM}{c^2\xi}$$

$$\xi = \theta D_d \Rightarrow \alpha \frac{D_s}{D_{ds}} = \frac{4GM}{c^2\theta D_d}$$

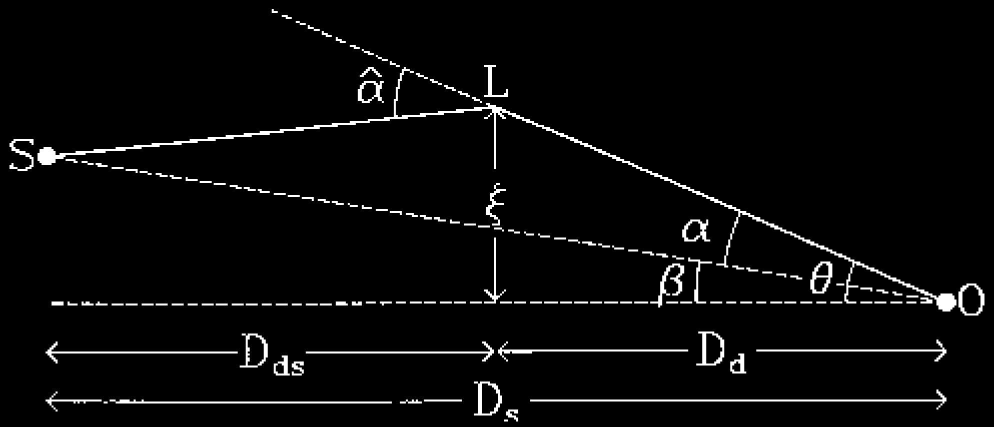
Gravitational Lens Optics



When the source and deflector (lens) are perfectly aligned, an Einstein ring is seen by the observer. In this case, α and θ are equal. This angle is called the Einstein angle, θ_E. Then

$$\alpha\theta = \theta_E^2 = \frac{4GM}{c^2}\frac{D_{ds}}{D_d D_s}$$

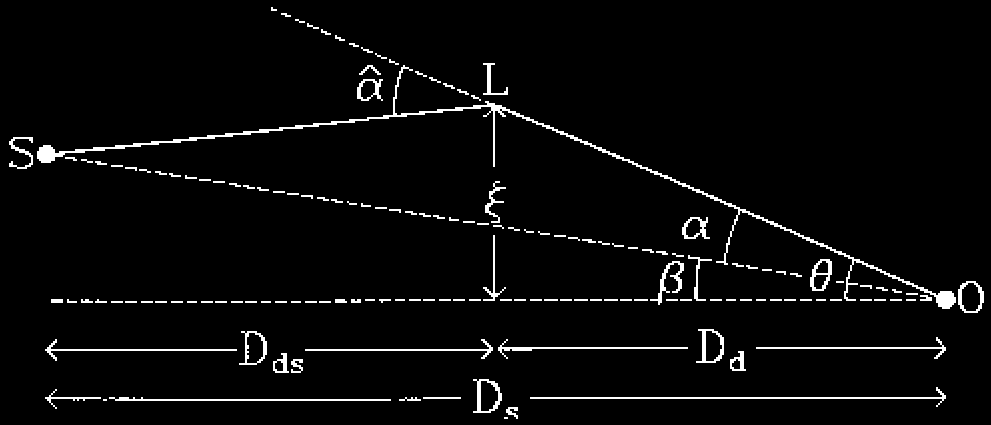
Gravitational Lens Optics



$$\alpha\theta = \theta_E^2 = \frac{4GM}{c^2}\frac{D_{ds}}{D_d D_s}$$

$\theta_E = \left(\frac{4GM}{c^2 D}\right)^{1/2}$, where $D \equiv \frac{D_d D_s}{D_{ds}}$ is the effective lens distance.

Gravitational Lens Optics

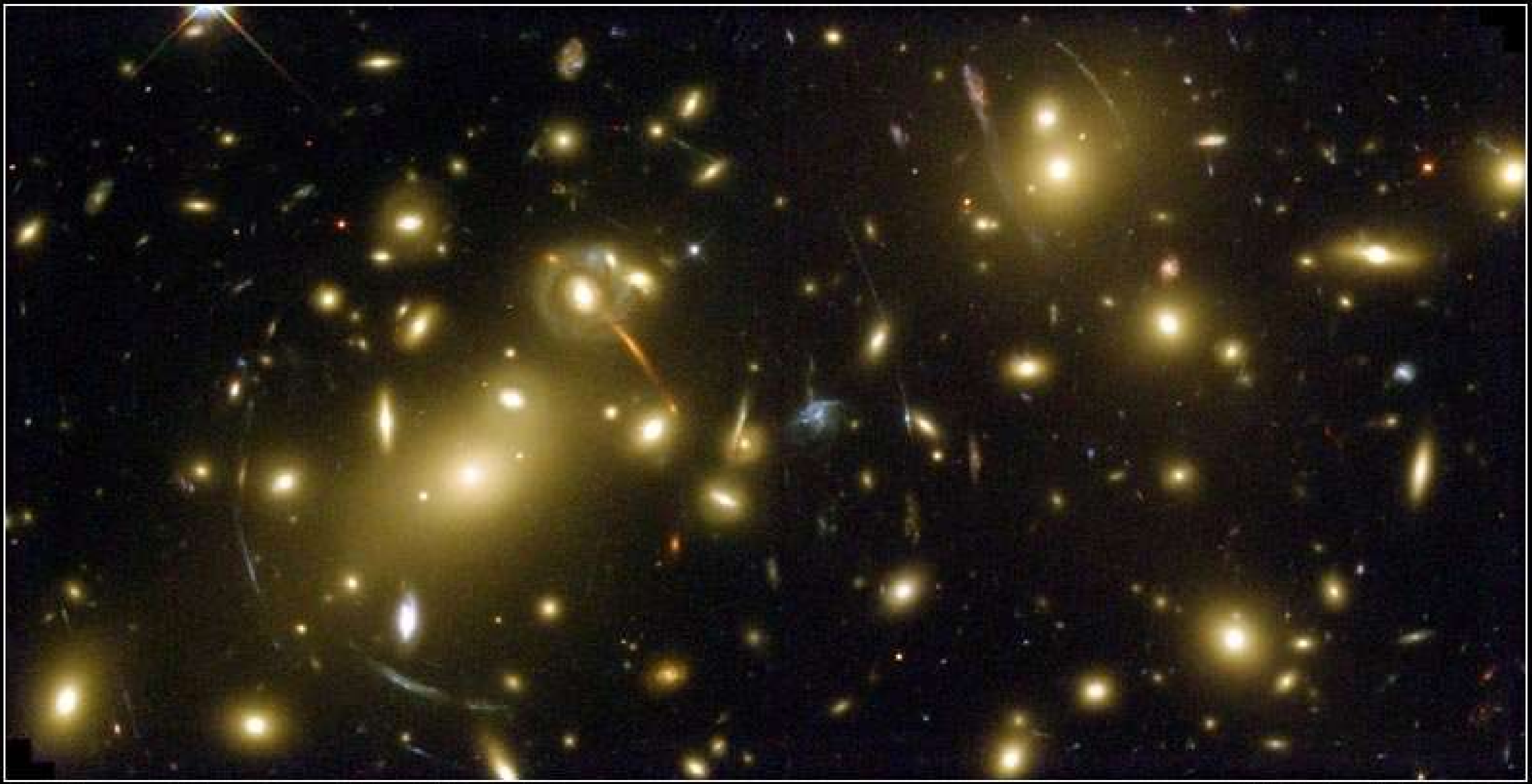


Expressing the mass in solar masses and the distance in Gpc leads to

$$\theta_E = \left(\frac{4GM}{c^2 D}\right)^{1/2} = 3\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{1\ \text{Gpc}}\right)^{-1/2} \mu\ \text{arcsec}$$

For example, we can estimate θ_E by measuring the radius of curvature of an arc. Then we can use this formula to estimate the mass of the lens “enclosed” by the ring.

$$\theta_E = \left(\frac{4GM}{c^2 D}\right)^{1/2} = 3\left(\frac{M}{M_\odot}\right)^{1/2}\left(\frac{D}{1\ \mathrm{Gpc}}\right)^{-1/2} \mu\ \mathrm{arc\,sec}$$



Galaxy Cluster Abell 2218 **HST • WFPC2**

NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

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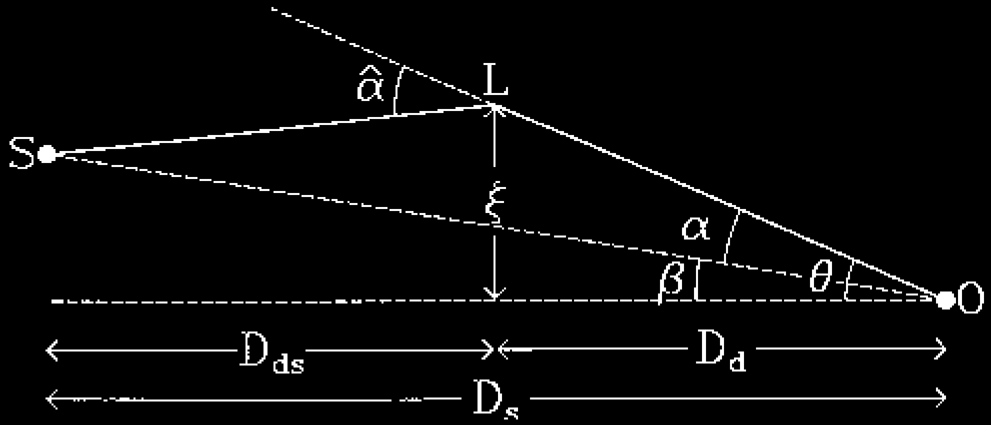
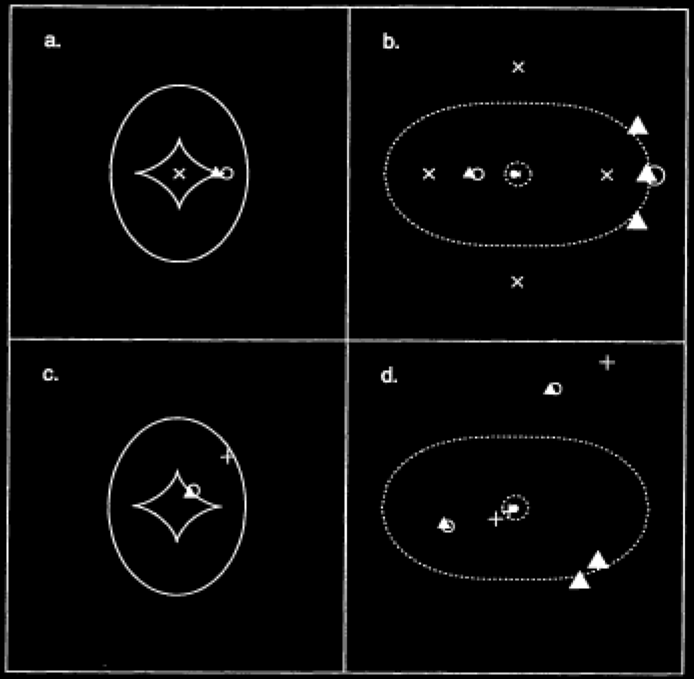


$$\boxed{\theta_E = \frac{4\pi\sigma^2 D_{ds}}{c^2 D_s}}$$

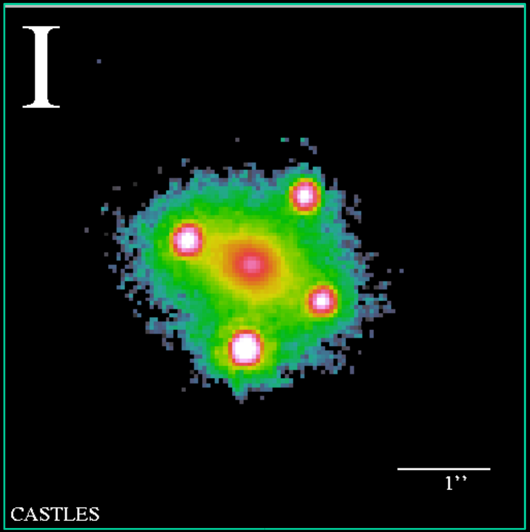
The above formula relates θ_E to the velocity dispersion σ.

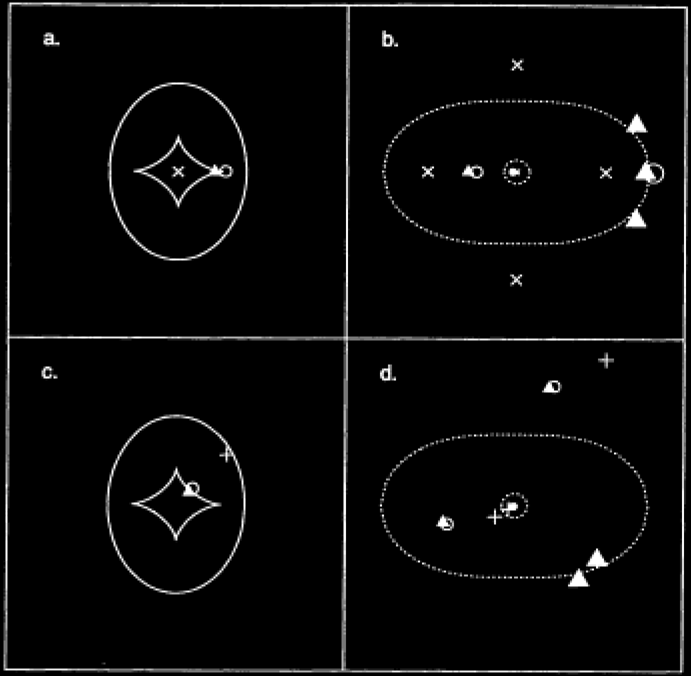
Caustics and Critical Curves

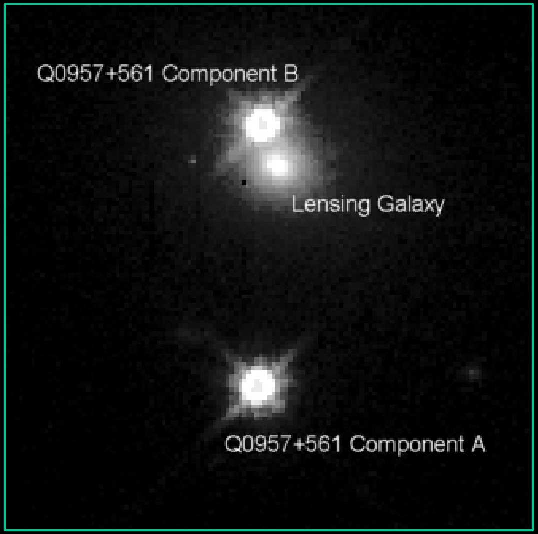


Left panels: point source positions, with caustic lines
Right panels: image locations, with critical curves
Top panels: Xs mark location of Einstein cross
Bottom panels: Q0957+561 midway between O and +
Image multiplicities: 1, 3 and 5 (weak image near center)

Caustics and Critical Curves







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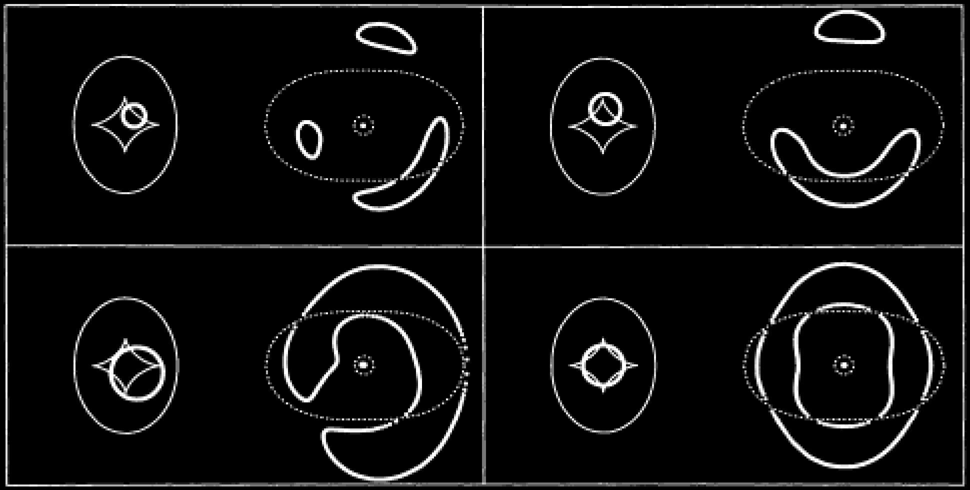
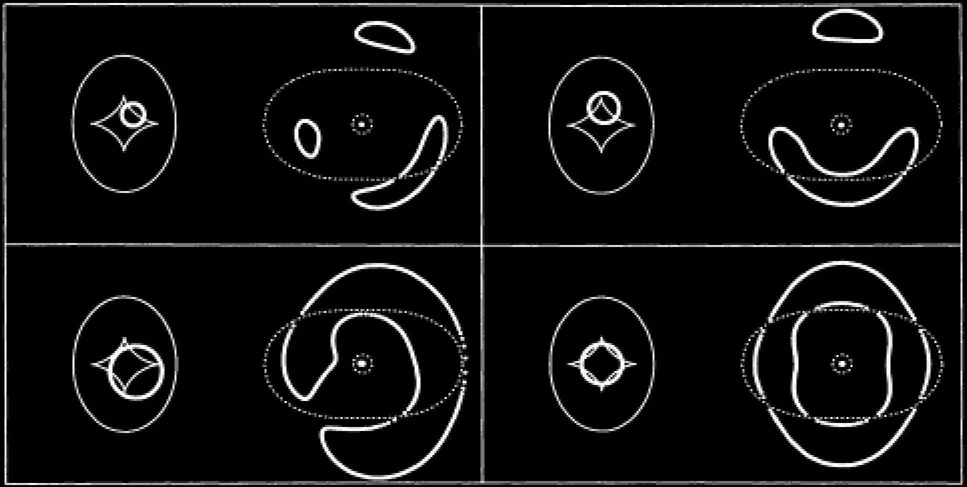
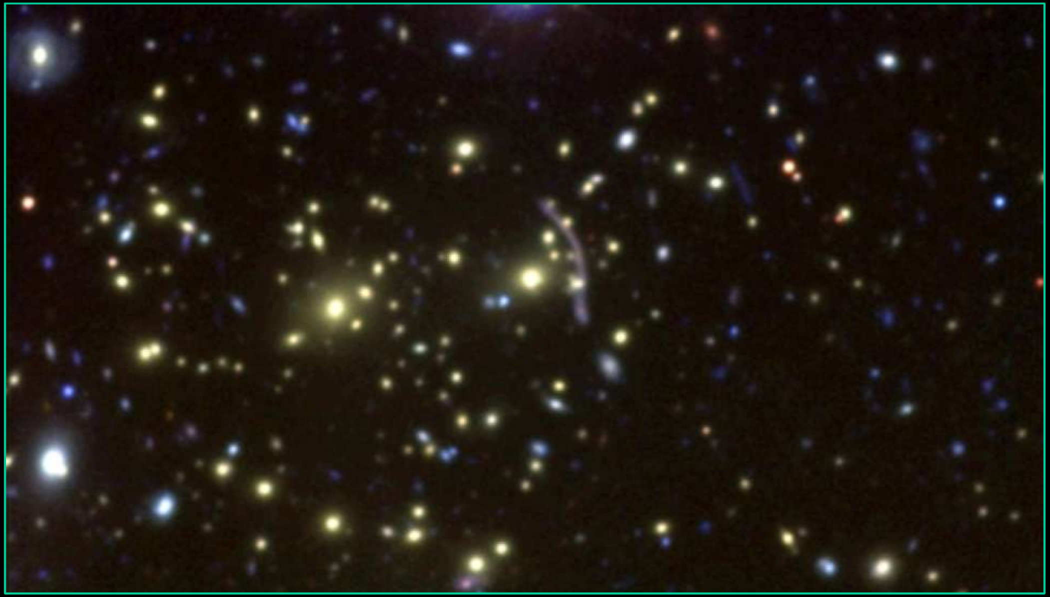


Arc and ring images of resolved sources. Each set contains source plane on left and corresponding images on right.

Top right set: Abell 370.
Bottom left set: MG1131+0456.

Caustics and Critical Curves

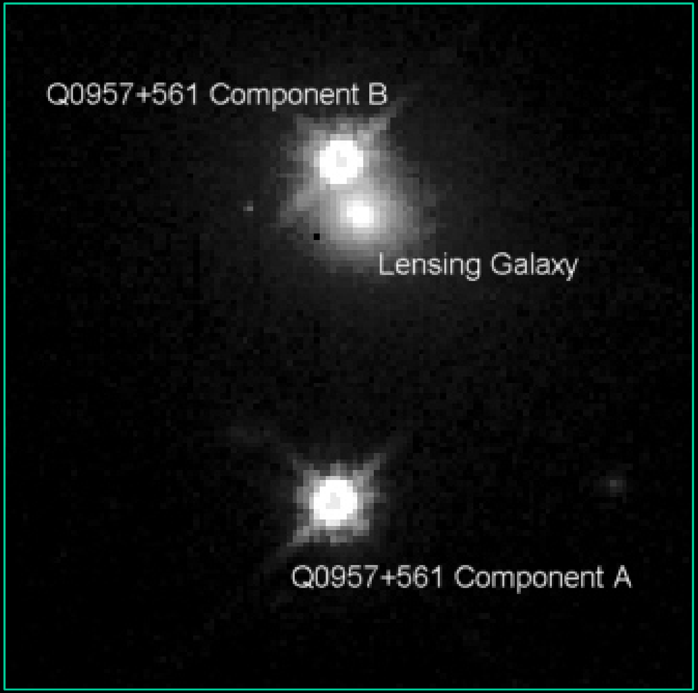




Applications of Gravitational Lenses

Cosmography - large scale geometry of universe

Hubble constant can be estimated by measuring the time delay between the two images of Q0957+561, a variable source



Schild 1990 $\Delta t = 1.1$ yr

Others (incorrect) $\Delta t = 1.48$ yr

Falco et al model (1991a):

$\Delta t = 1.48$ yr, $\Omega_0 = 1$, $q_0 = 1/2$

$\Rightarrow H_0 = 61 \pm 7$

Applications of Gravitational Lenses

Cosmography - large scale geometry of universe

Redshift - distance relation: gravitational lenses confirm that high redshift sources lie behind lower redshift lenses

Applications of Gravitational Lenses

Dark Matter Studies - Our Galaxy

MACHOs? (MAssive Compact Halo Objects)

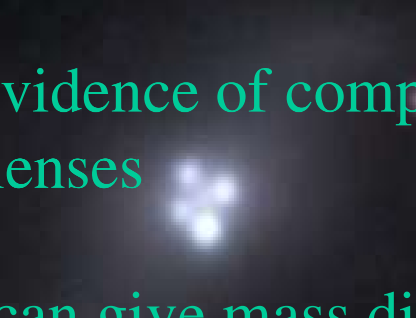
Examples: "comets," "asteroids," "Jupiters," brown dwarfs, cool white dwarfs, neutron stars, black holes

Microlensing - when a star in the LMC crosses a lens caustic, it will briefly brighten (flux amplification)

Ryden text: research indicates as much as 20% of halo mass could be MACHOs. Typical mass > 0.15 solar masses, perhaps cool white dwarfs.

Applications of Gravitational Lenses

Dark Matter Studies - External Galaxies



Lensing galaxies - estimate mass, M/L ratio

Schneider et al 1988 - not much dark matter in galactic cores

Narasimha et al 1986 - evidence of compact masses ~ 10^{10} solar masses in center of lenses

Galactic halos - models can give mass distribution as well as mass of lenses

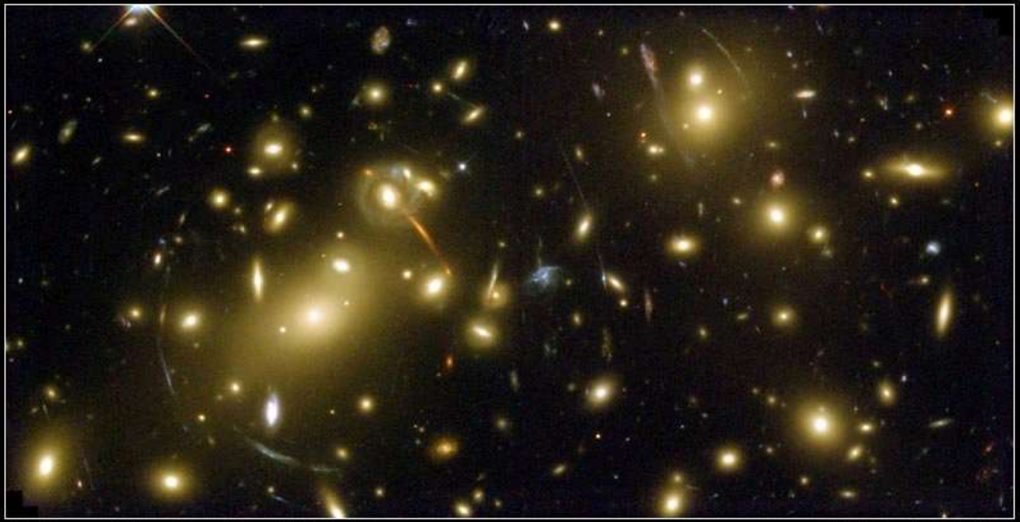
Existence of lens candidates with no detected lensing galaxy - possibility of galaxy-sized condensations of dark matter

Applications of Gravitational Lenses

Dark Matter Studies - Galaxy Clusters

Advantages of lensing clusters vs. lensing galaxies:

- larger angular area of sky, so greater cross-section for lensing
- sources are faint galaxies, more numerous than quasars
- sources are resolved, not point-like
- can study mass distribution in clusters



Galaxy Cluster Abell 2218 **HST** • WFPC2

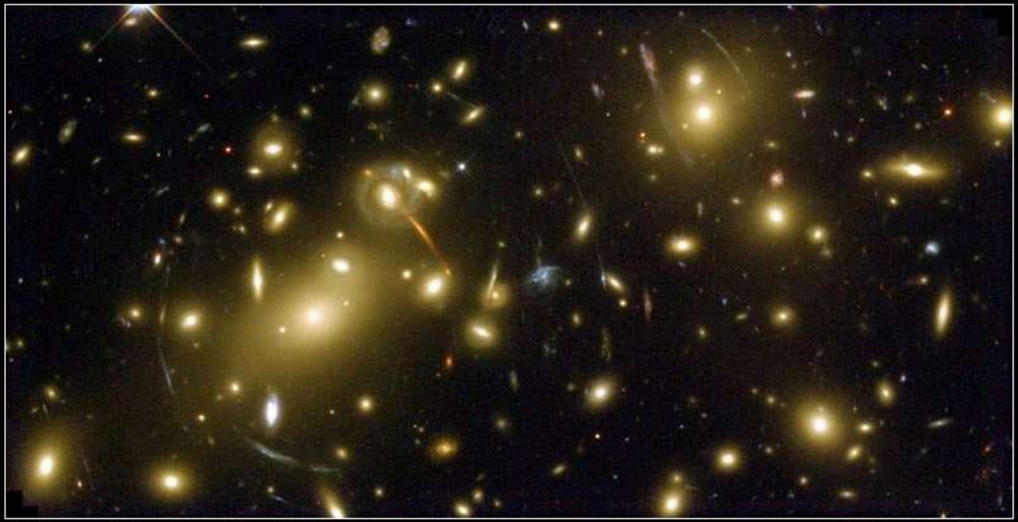
NASA, A. Fruchter and the ERO Team (STScI) • STScI-PRC00-08

Applications of Gravitational Lenses

Dark Matter Studies - Galaxy Clusters

Mass models can estimate velocity dispersion, M/L ratio, core radius, ellipticity, mass distribution

Arc radius of curvature ~ $\theta_E \Rightarrow$ estimate σ, M/L ratio



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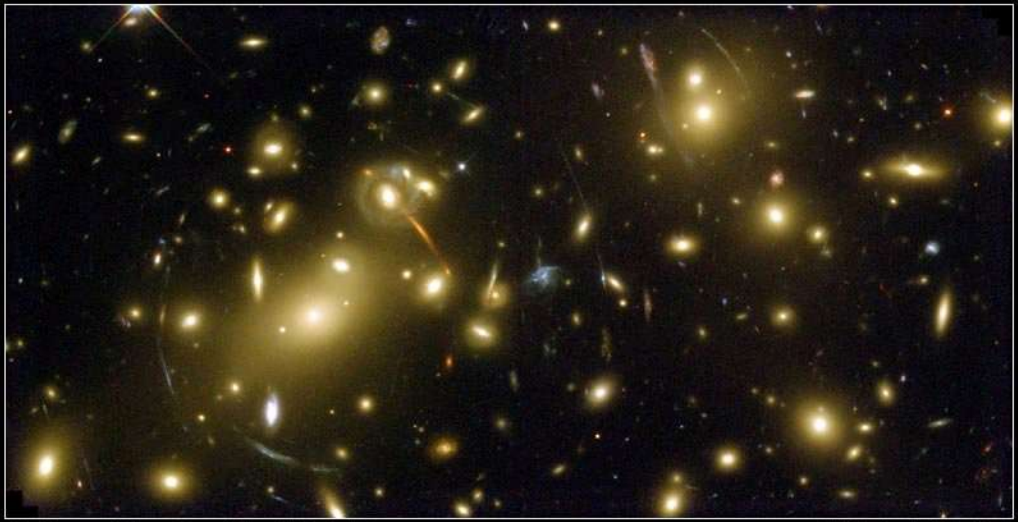
Applications of Gravitational Lenses

Dark Matter Studies - Galaxy Clusters

Distribution of arclets gives information on mass distribution, therefore information on dark matter distribution in lensing cluster

Source near caustic ⇒ large magnification (tangential elongation)



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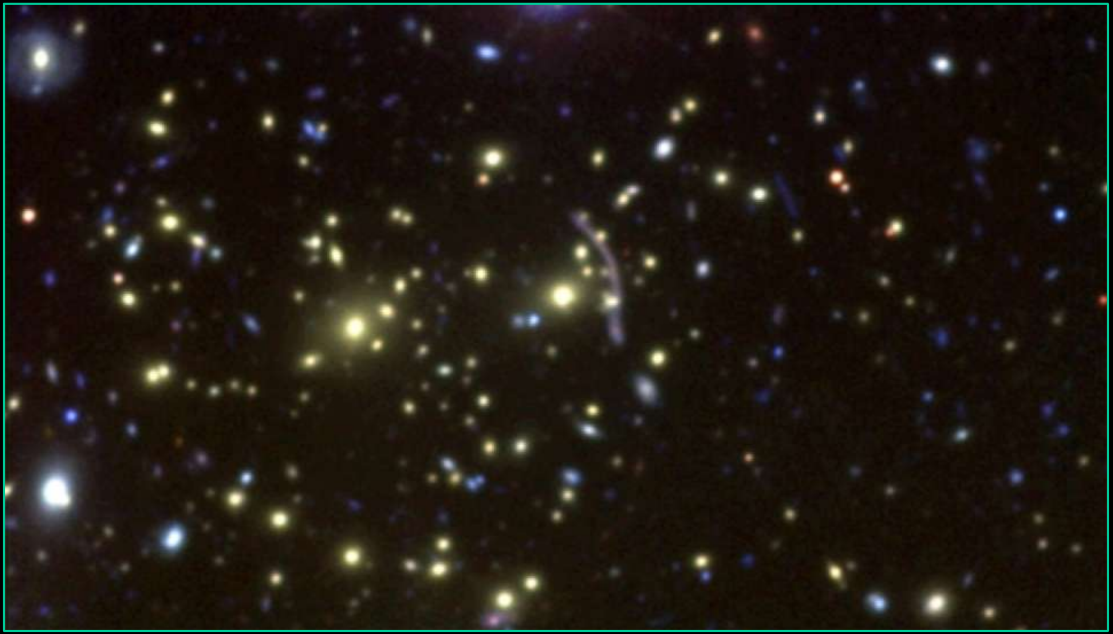
Applications of Gravitational Lenses

Dark Matter Studies - Galaxy Clusters

Findings:

- M/L ≥ 100 solar units
- core radius < 100 kpc (smaller than their images)
- Abell 370 appears to have elliptical mass distribution



Applications of Gravitational Lenses

Dark Matter Studies - Large Scale Structure

Inhomogeneities up to 50 - 100 Mpc ⇒ elliptical distortions in images of distant sources and changes in apparent luminosities

Discovery of faint blue galaxy population provides a potential way to study these effects

If cold dark matter cosmology is correct, should see 1 - 3 % ellipticity

Possibility of studying lensing by cosmic strings

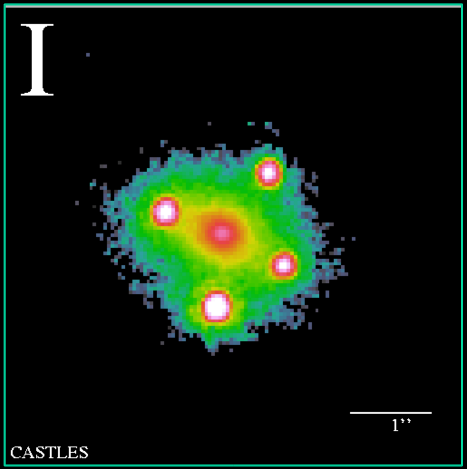
Applications of Gravitational Lenses

<u>Magnification of Very Distant Objects</u> - <u>Quasars</u>

Microlensing ⇒ constraints on quasar size, because the flux variation is significant only if angular size of source is less than Einstein radius of lens.

Q2237+030 (Einstein cross) - first direct estimate of quasar size

Size < 10^{10} km, but inconsistent with accretion disk model (nonthermal component?)

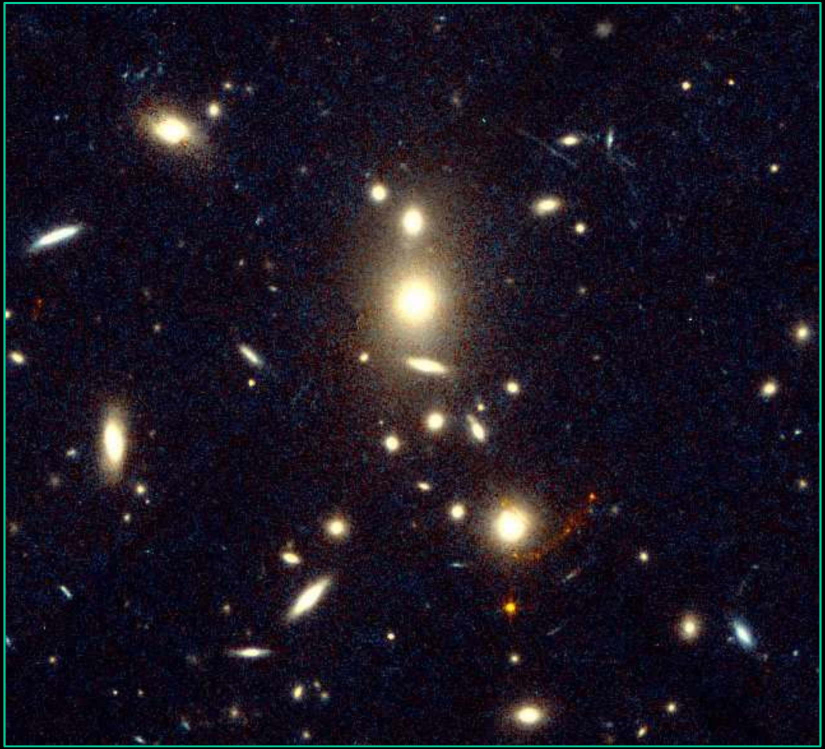


Applications of Gravitational Lenses

<u>Magnification of Very Distant Objects</u> - <u>Galaxies, Radio Sources</u>

Resolved sources - high resolution information from parts of the source near caustic (high magnification)

Spectroscopy possible at 3 magnitudes fainter than normal, so arc redshifts can be determined

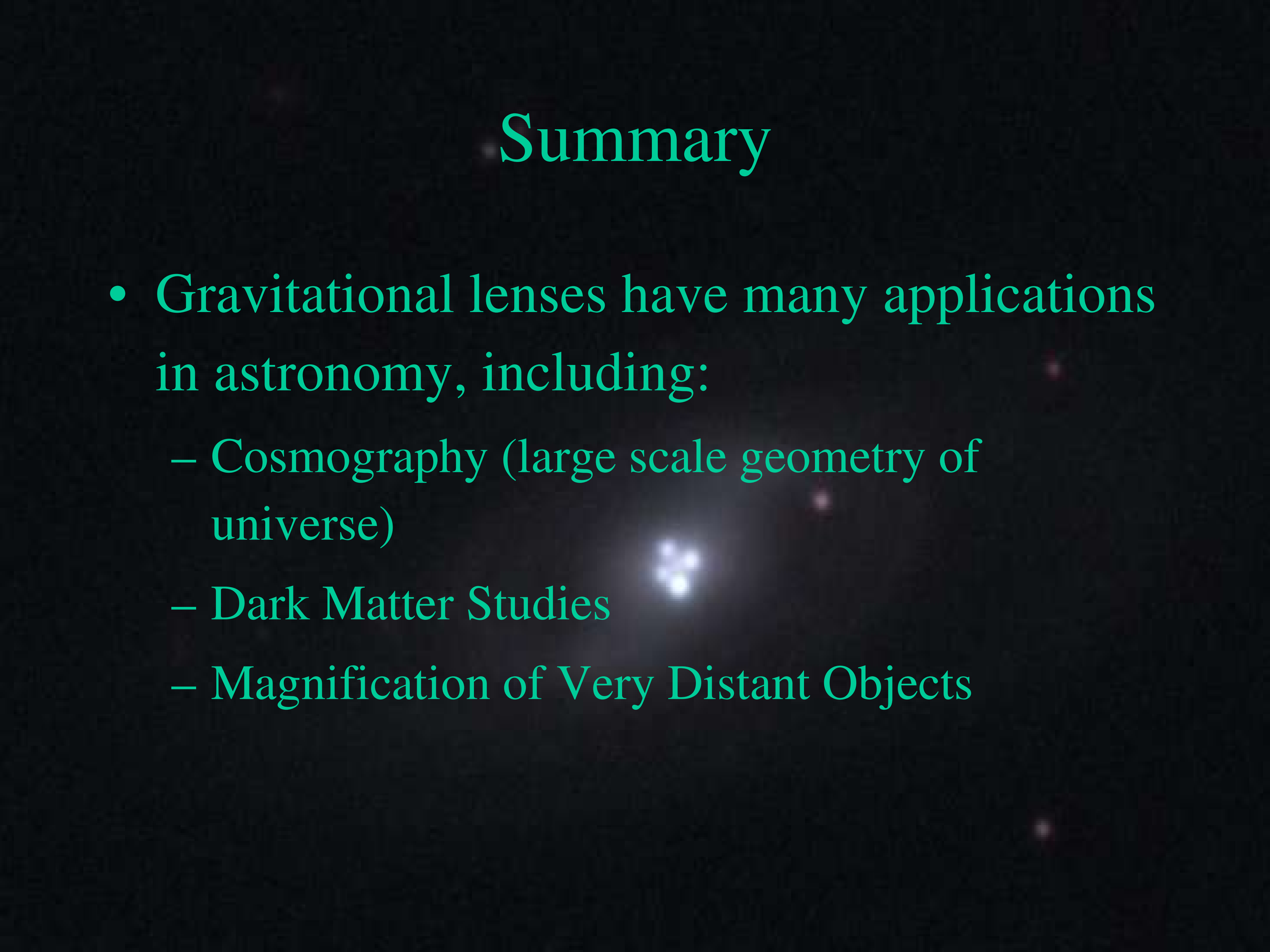


CL1358+62 (HST image)

Lens: galaxy cluster, $z = 0.33$

Source: galaxy pair, $z = 4.92$

Summary



- Gravitational lenses have many applications in astronomy, including:
  - Cosmography (large scale geometry of universe)
  - Dark Matter Studies
  - Magnification of Very Distant Objects

Thanks, everybody!