Project Overview

In this project, you will explore the 20th century discovery that the Universe is expanding, or similarly, you will find that the further away an object is, the rate at which it appears to move away from the observer increases. In effect, you will explore the phenomenon of cosmological redshifts, using the Hubble Ultra Deep Field, which contains thousands of galaxies, many of which have had their redshifts, stellar composition, and ages determined. You will use this dataset, in combination with the principle of the Doppler Shift of light, to determine the rate of this expansion, which is named in honor of the American astronomer Edwin Hubble who first discovered it. Upon completing this exercise, you will have explored the most significant astrophysical “photograph” ever produced and develop your understanding of:

1) the Doppler Shift of Sound and Light
2) galaxy type, morphology, and the evidence for galaxy formation models
3) the Hubble Expansion of the Universe and the estimate to the age of Universe
A primer on the qualities of waves

The electromagnetic phenomena that we call “light” can be adequately understood by its wave-like properties. If you are unfamiliar with the physical description of waves, consider the following figure.

A wave can be fully defined by its wavelength and amplitude, which are outlined below.

A wave can similarly be described by its frequency. The definition of the frequency is: 
Frequency = 1 / Period. The definition of wavelength Period is simply the time it takes for the wave to cycle between peaks (or troughs, or any similar amplitude)

What do these terms mean physically? For sound waves, the relationship between wavelength/frequency and “pitch” of a wave:

HIGH “pitched” sounds have SMALL Wavelengths and HIGH Frequencies
LOW “pitched” sounds have LARGE wavelengths and LOW Frequencies

Light waves are a different physical phenomenon than sound waves. They are manifestations of oscillations in electro-magnetic fields, whereas sound waves are pressures disturbances in a dense medium. For light waves, the relationship between wavelength/frequency and “color” of a wave is:

REDDER light has LONGER Wavelengths and LOWER Frequencies
BLUER light has SHORTER wavelengths and HIGHER Frequencies

Christian Doppler observed, in the 1800s, that the pitch of a sound emitted from a source that is moving with respect to the observer varies as a function of the moving object's velocity. This is a phenomena you are all familiar with, if you have ever noted the change in the sound of a firetruck's or police car's siren that occurs as the sound sources moves away or towards you at a high velocity. This fundamental physics phenomenon is, in honor of its discover, called the Doppler Shift of Sound.
Because light can be described by its wave properties, you might expect that the Doppler Shift phenomenon would exist. In fact, it does and the magnitude of the Effect can be quantified for each of these phenomena with a simple equation, provided in the Appendix.

Now for some Astrophysics.

Rather than referring the observed shifts in wavelength, astronomers simply refer to the observed shift by the color variation. You can use the table below as a reference:

<table>
<thead>
<tr>
<th>If the light is....</th>
<th>...then the emitter is....</th>
<th>...and we call this effect a</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Bluer than expected”</td>
<td>moving towards us</td>
<td>“blueshift”</td>
</tr>
<tr>
<td>“Redder than expected”</td>
<td>moving away from us</td>
<td>“redshift”</td>
</tr>
</tbody>
</table>

In a mildly confusing abbreviation, astronomers have developed a short hand notation, $z$, called the “redshift” parameter. This is just a definition, and as such, is exact.

Equation 1:

$$z = \frac{\lambda - \lambda_0}{\lambda_0} \Rightarrow \lambda = \lambda_0(1 + z)$$

Making this substitution for the red-shift parameter, the Relativistic Doppler Equation can be rewritten in a form that is most familiar to astronomers.

Equation 2:

$$1 + z = \left( \frac{c + v_0}{c - v_0} \right)^{1/2}$$

where $c = 2.99 \times 10^8$ meters per second, the speed of light in a vacuum.

For source velocities much much less than $c$, we can appropriately define $z=c/v$. But, when considering light from extremely distant objects, though, you cannot use this approximate to quickly determine the redshift velocity. Instead, you have to use the following equation to determine the recessional velocity, which is a simply a different representation of Equation 2.

Equation 3:

$$v_0 = \frac{(1 + z)^2 - 1}{(1 + z)^2 + 1}$$

But, a word of caution! This equation could be completely meaningless depending on the object (e.g. galaxy or star) that is observed. The reason for this arises from the both the definition of a “velocity” and the underlying physical process which gives rise to an observed redshift.
For redshifts determined for stars, or planets or nearby (i.e. interior to our own Milky Way galaxy) objects, this equation provides a valid estimate to the velocity of that object. In this scenario, the observed object is physically moving through space, that is to say its physical position “Δx” changes over time “Δt” in a particular direction (e.g. towards or away from the observer) which fits the definition of a velocity.

In the case of the distant galaxies, though, the observed redshift occurs not because these galaxies “move” away from the observer but rather because these galaxies reside in a Universe that is itself expanding (and in fact accelerating in this expansion) and this expansion itself red-shifts the light. Spacetime itself is “stretching” out over time and this causes the light from distant objects to be red-shifted. That is, the length of your ruler, measured across cosmic times increase with time and so correspondingly, the wavelength of light emitted over cosmic distances itself increases. As progressively longer wavelengths correspond to progressively redder “colors” the light that we observe from distant galaxies is redder than “expected” if it were emitted in the viewer’s rest frame. These red-shifts are often referred to as cosmological red-shifts and do not indicate true velocities. As a result, equation 3 is aphysical by definition for these redshifts. If the Universe were “standing still,” there would be no cosmological red-shift measured for distant galaxies, though they very well could display redshifts resulting from their true motion in space.

**Redshifts in Practice**

Now, that we have an understanding of the mathematics behind the Doppler Shift of light, how do astronomers figure out what the specific emission from a distant object is telling them about the Universe? A spectrometer will “break apart” light into its components. White light is actually composed of all the colors of the rainbow (Red, Orange, Green, Blue, etc.) each the manifestation of photons with varying wavelengths. So too is light from composed of a variety of the (superposition of) wavelengths. Thus, a spectrometer attached to the business end of a telescope can be used to understand the nature of objects for observed celestial objects.

Consider the spectrum below, taken of a star near our Sun.
The vertical axis is the flux (measured in units of energy per time per area) of the light as it is received by the telescope; the x axis is the wavelength of the light, and moving from the right to the left you are observing light that is more and more “red”. The depressions and peaks which appear throughout the spectrum are absorption and emission lines. These variations occur in the spectrum when an atomic species in the star's atmosphere absorbs (or emits, hence the nomenclature) a photon (a particle of light) of a particular energy. We can predict the wavelengths at which these features should appear for a given system of atoms, in the laboratory.

Consider that an absorption line at 6366 Å was predicted by laboratory experiments to observed be at 6364 Å. How fast must the star be physically moving with respect to the observer at rest, to induce such a redshift?

To solve this problem, begin by writing down what you know (note that 1 x 10^{-8} cm = 1 Å):

What is \( \lambda_0 \)? ____________________ in cm? ____________________________________
What is \( \lambda \)? ____________________ in cm? ____________________________________

Is this object moving towards or away from the Earth? That is, is the observed wavelength longer (redder) or shorter (bluer) than expected?

_______________________________________________________________________
_______________________________________________________________________

Using the definition of the red-shift, that is:

\[
z = \frac{\lambda - \lambda_0}{\lambda_0}
\]

how much has this light been red-shifted/blue-shifted?

What does \( v \) equal in meters per second? Here, because the redshift is small, you can use the relation \( v = cz \).

How fast is this object moving towards or away from the Earth in centimeters per second?
Last, but not least, if 55 centimeters per second is roughly equivalent to 1 mile per hour, how fast is this star moving away, in miles per hour?

The HUDF as an astrophysical tool

In the following sections, you will explore the idea of an expansion Universe. To do this, you will use one of most tantalizing images ever taken with a telescope. This image is called the Hubble Ultra Deep Field (HUDF) and it was produced using the Hubble Space Telescope. After the HUDF data were collected, ASU students and professors led by Dr. Rogier Windhorst, Matt Mechtley, and Lisa Will developed a Java program that allows a person to “zoom” through space and time to gain a better understanding of the distant Universe. This package is called “Appreciating Hubble at Hyperspeed,” and you will use this software package to understand this expansion of the Universe.

Introduction to AHaH:

For this exercise you should choose one partner. In the next few minutes, you and your partner should familiarize yourself with the AHaH software. Click on the objects that you see in the image, zoom around, and move around in the image until you feel comfortable with the layout of the program. A mouse and keyboard is all that is necessary to use this software package. The mouse functions are provided here:

```
Movement:
- To move side-to-side, use the Left Mouse button or Arrow Keys
- To move forward or backward in space, use the Mouse Wheel, Middle Mouse button, or Shift-Arrow Keys
- To move automatically to an object, double left click on the object

Information and Display:
- To jump to a specific object or redshift, press J
- To obtain more information about an object, left click on the object

- To toggle between the Universe's real geometry and a geometry where angular sizes are unaffected by the Universe's expansion, press G
- To reset the view to the origin, press R
```
If you find an interesting galaxy, and would like to see more information about it, you can left click on that object to bring up a screen that resembles the one below.

This information lists a few important points regarding each galaxy. We will directly use the first three in the following exercises. The first quantity is the Object ID, which is an arbitrary number assigned to each of the galaxies in the HUDF/AHaH field, and should be copied for each galaxy you observe. The second quantity is the redshift. This is equal to $z$ in the aforementioned equations, and is a unitless measurement of the redshift. The third important quantity in this infobox is the Co-moving Radial Distance, or CRD. This value corresponds to the distance along the line of sight to the objects. Here, distances are provided in units of Mega-parsecs, a unit which is roughly equivalent to ~3 Million lightyears, or 100 Billion Astronomical Units.

For those interested, the CRD equation is provided in the appendix to this exercise. Suffice it to say that the geometry of the Universe is dependent a number of constants that have been well-constrained by observation. In the AHaH software package, the Concordance Model is assumed, but you can change these parameters and explore how varying the relative densities of objects in the Universe can change this distance measure.

**Exercise 1: Galaxy morphology over cosmic time**

In this exercise you will consider a variety of galaxies over a range of redshifts in order to derive any morphological trends which may arise in the HUDF, over cosmic timescales. Though it is a far from perfect classification system, the morphology (or in other words, “the way galaxies look”) of a galaxy can often provide a unique insight into the processes ongoing in galaxies. In general, we find that the broadband morphologies of galaxies fall into one of three categories—spiral, elliptical, and irregulars:

**-Spirals:** Spiral galaxies are probably the most familiar class of galaxy. The Milky Way galaxy is an example of a spiral galaxy. They are easily identified by their discs, in which knotty, clumpy star clusters and spiral “arms” may be found. In some galaxies, the arms are quite diffuse and not “tightly wound,” but these galaxies may still be typed as Spirals. In the center of these galaxies, one can typically find a bright “bulge” of concentrated gas, dust and stars. There are two major subclasses of Spiral galaxies, those with and those without a pronounced bar-structure. Spiral galaxies exhibiting a bar structure are given the designation SBx, where “x” is some letter a, b, or c, or some combination thereof. Spirals without bars are denoted Sx, with the same definition of “x” as before. Because these galaxies are often actively forming stars, they appear “bluer” in color.

**-Ellipticals:** As the name would suggest, these galaxies are spherical or elliptical in shape. They lack the “arms” found in Spirals, and often look like reddish footballs, spheres, or cigars. The redness of these galaxies in the local Universe (I.e. at $z \ll 1$) can be attributed to starlight from old-aged stars, comparable in age (or older) to the age of
our Sun. A word of caution, though: Don't label every “red” object in this field an elliptical. The galaxies in the HUDF are at high redshift, and as a result their light is redder in the observer's frame. This means that for high redshift galaxies, though you are observing in at optical wavelengths, the emission you observe may have been emitted at much shorter (blue Visible or even Ultraviolet) wavelengths.

**Irregulars:** Some galaxies can not be classified neatly as Spiral or Elliptical (or some variation thereof). These galaxies are then lumped into a catch-all morphological class designated “Irregular.” There are few similarities in morphology between each of these galaxies in this class: they have a wide range of masses, stellar populations, structures and shapes. Many irregulars, though, can be described as “disturbed.” Within dense galaxy cluster environments, the passage of galaxies near to other massive galaxies, or the collisions between two galaxies, can dramatically disrupt galaxies which could may have previously been well-classified as Spirals or Ellipticals.

Pictured below is a classification scheme, the Hubble Tuning Fork Diagram, which is not motivated by physics, but is used traditionally to outline the various morphology classes. One can find ellipticals not pictured here in the HUDF. These galaxies can be designated E$n$, where $n$ is some integer value between 1 and 7. Similarly, Spirals may not be nicely categorized either. For example, a tightly wound spiral galaxy, without a bar and devoid of a dusty, gassy inter-arm region may be adequately classed as an “Sab” or “Sa/b”. Galaxies with less tightly wound spiral arms are labeled Sb, Sc or sometimes Sd.

Now, using the AHaH software classify at least 5 galaxies per each of the following redshift intervals. Click on each of the galaxies that you classify, and also make a note of each galaxy's ObjID, Redshift, and CRD and record these in the table on the next page.
<table>
<thead>
<tr>
<th>0.0 &lt; ( z ) &lt; 0.5</th>
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</thead>
<tbody>
<tr>
<td>ObjID</td>
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<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>0.5 &lt; ( z ) &lt; 1.5</td>
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<td>----------------------</td>
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<td>ObjID</td>
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<tr>
<td></td>
</tr>
<tr>
<td>1.5 &lt; ( z ) &lt; 4.0</td>
</tr>
<tr>
<td>----------------------</td>
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<tr>
<td>ObjID</td>
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<td>( z ) &gt; 4.0</td>
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<td>ObjID</td>
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</table>
Interpreting your results:

For the entire sample set:

What percent of the galaxies in your sample were Spirals?_________________________
What percent of the galaxies in your sample were Ellipticals?_______________________
What percent of the galaxies in your sample were Irregulars?_______________________

For galaxies at a redshift less than 1.5:

What percent of the galaxies in your sample were Spirals?_________________________
What percent of the galaxies in your sample were Ellipticals?_______________________
What percent of the galaxies in your sample were Irregulars?_______________________

For galaxies at a redshift greater than 1.5:

What percent of the galaxies in your sample were Spirals?_________________________
What percent of the galaxies in your sample were Ellipticals?_______________________
What percent of the galaxies in your sample were Irregulars?_______________________

Do you notice any change in the number density of galaxies as a function of redshift? If so what sort of change?
What you have likely discovered is that the number density of irregular galaxies or spiral galaxies increased as a function of redshift, or similarly that the density of elliptical galaxy decreased. This is not a trivial result and was one of the Hubble Telescope's major discoveries in the 1990s.

In the simplest analogy of hierarchical assembly model, smaller galaxies (like spirals) in high density environments merge or assemble to form more massive bulges and potentially elliptical galaxies. Thus one would expect that as you look to greater redshifts, you would find a higher number density of spirals or merger remnants. There is one caveat to the analysis you have just conducted, which has been alluded to already. As you explore galaxy morphology at higher redshifts, you move out of the visible rest-frame and into the UV rest-frame. In this rest-frame, galaxies can display dramatically different morphologies, and spiral galaxies in particular can begin to display irregular-type morphologies. So, some of the galaxies which you identified as irregular morphology may, in fact, be classed otherwise if they were at low redshifts.

In spite of this, the morphologies of galaxies in the HUDF and similar deep field surveys do seem to support the hierarchical assembly theory.

Exercise 2: Calculation of the Hubble Law for the Expansion of the Universe

In this exercise you will derive the Hubble Law, so named for Edwin Hubble the scientist who discovered it in 1929. This single insightful calculation single-handedly re-defined modern astrophysics and cosmology by giving tangible evidence for the Hot Big Bang model that was proposed by the Belgian priest George Lemaitre. This model, in its modern iteration, proposes that the Universe began approximately 14 billion years ago in an extremely energetic expansion event. It has been consistently re-confirmed by a wide variety of independent studies, and we now roughly understand the evolution of the Universe from the present day back to roughly $10^{-43}$ seconds after the birth of the Universe.

To begin this exercise, identify an additional 10 galaxies with redshifts $z < 1.0$. Add these to the following table. To determine the velocities of these objects use Equation 3.
The Hubble Law defines the expansion rate of the Universe. Thus, we can use the straightforward rate equation to determine the magnitude of this expansion.

\[ Rate = \frac{Velocity}{Distance} \]

You should not convert your redshifts in the table above to velocity. These numbers should all be less than one, and you can use an equation defined previously in this exercise. We will leave them in terms of the speed of light for now, and make the conversion at the conclusion of this exercise. To determine the Hubble Relation:

1) On graph paper, for each of the \( z < 1.0 \) redshift galaxies, plot on the vertical axis the velocity of the galaxy with its corresponding CRD on the horizontal axis.

2) Calculate the slope of the best fit line to these data points. You should use a straightedge, and you can minimize the error of the fit by eye.

3) Determine the magnitude of this slope. Convert this slope into the units of km/s/Mpc by multiplying by the speed of light in the appropriate units. This constant is called the Hubble Constant, \( H_0 \), and it relates the velocity of a distant galaxy to its line of distance by the following equation

\[ v = H_0 d \]

What is the sign and value for \( H_0 \): __________________________________________

Which galaxies are moving at greater velocity with respect to the Earth, the more or less distant galaxies?_________________________________________________________

If the accepted magnitude of the Hubble's Constant is 70.1 km/s/Mpc what is the percent error in your estimation. You can determine the percent error with the following formula:

\[ P.D. = \left| \frac{Calculated - Actual}{Actual} \right| \times 100\% \]

Report your error here:
Lastly, because the slope of this line effectively has units of time (here, seconds), one can make a first approximation to the age of the Universe, assuming a Hubble rate of Expansion.

To make this estimate, you simply need to convert the units of $H_0$ to units of time, explicitly. Note that 1 kilometer equals to $1 \times 10^5$ cm and 1 Mpc equals to $3.08 \times 10^{24}$ cm. Once you have reduced the units of this constant to s$^{-1}$ you need only to invert the Constant.

Report your estimate to the age of the Universe here:

Note that 1 yr equals to $3.15 \times 10^7$ seconds. What is your estimate to the age of the Universe in years?

If the accepted age of the Universe is approximately 13.7 billion years, what is the percent error between your calculated age and the accepted age?

Why the great disparity between ages? What does this imply about the expansion of the Universe over the entire lifetime of the Universe?
The appendices below are not necessary for this lab exercise, but they are provided to help you to better understand a few specific phenomena discussed in the text.

**Appendix A: Doppler Equations for Light and Sound**

For Sound:

\[ \lambda = \lambda_o \left(1 + \frac{v}{v_c}\right) \]

For Light:

\[ \lambda = \lambda_o \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} \]

Here, the subscript “o” indicates “the source reference frame” and, \( v \), is the velocity of the object with respect to the observer. You’ll note that, though these equations are similar in form, they do differ. In short, this discrepancy arises from the physicist Albert Einstein's theory of Special Relativity, and as a result we typically refer to the Doppler Shift for light as a relativistic Doppler Shift. The crux of Special Relativity is that, unlike the speed of sound, the speed of light constant in all inertial frames of reference and is usually abbreviated as \( c \).

**Appendix B: A brief note on the Co-moving Radial Distance**

The CRD is a cosmological distance measure which can be derived from Einstein's General Relativity field equations, a derivation first made by Alexander Friedman and George Lamaitre in the 1920s. Cosmological distance measurements are fundamentally linked to the composition of the Universe (e.g. energy, matter, radiation etc.). Different compositions yield different “curvatures”, which dictate the metric by which universal geometries can be derived.

In layman’s terms, the density of “stuff” in the Universe (e.g. atoms, dark energy, or WIMPs) defines the physical Universe. Each component of the Universe affects this geometry differently, and typically the density of each of these components is represented by the greek Omega as the ratio of the component density with respect to the “critical density.” The critical density is a special case gleaned from the reduced F-L expansion equation which, without going into the details of the equation itself, corresponds to a Universe which continues on without accelerating in its expansion (if it has an initial expansion velocity at all).

The CRD equation can be calculated using the following formula:

\[ D_c = D_H \int_0^{z'} \frac{dz'}{E(z')} \]

where \( D_H \) is the Hubble Distance, \( c/H_o \), and \( E(z') \) equals to

\[ E(z') = \sqrt{\Omega_0 (1 + z)^3 + \Omega_k (1 + z)^2 + \Omega_\Lambda} \]

and

with \( G \) equal to the constant of Universal Gravitation, \( H_o \) the Hubble Constant, \( \Lambda \) the Cosmological Constant, \( k \) the curvature scalar.